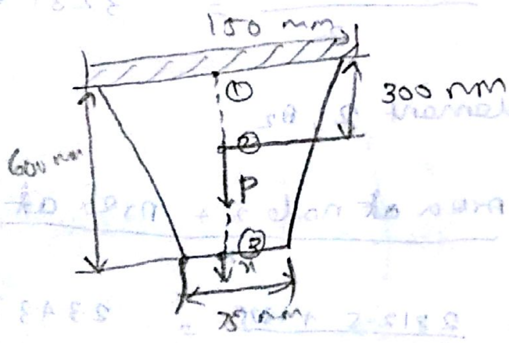
(5) consider a tapper steel plate of uniform thickness $t = 25$ mm as shown in fig. the young's modulus of the plate $E = 2\times10^5$ N/mm² and weight density $\rho = 0.82\times10^{-4}$ N/mm³. In addition to its self weight. The plate is subjected to a point load $P = 100$ N at its mid point. calculate the following By modelling the plate with 2 finite elements (i) Global force vector (ii) Global stiffness matrix. (iii) displacement in each element (iv) stress in each element (v) Reation force at its support



Sol:-

Area at node 1 A_1 = width × thickness

$= 150 \times 25$

$A_1 = 3750$ mm²

Area at node 2 A_2 = ~~width ×~~ $\frac{w_1 + w_3}{2} \times$

$= \frac{w_1 + w_3}{2} \times$ thickness

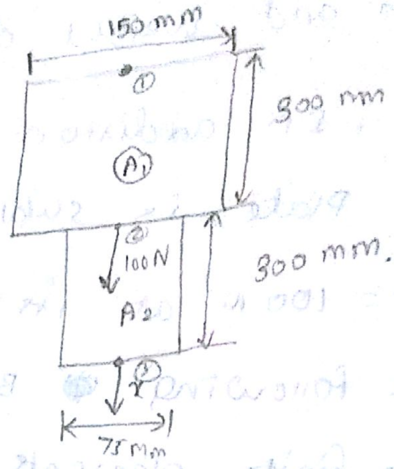
$= 2812.5$ mm²

=

Area at node 3, A_3 = width x thickness

$= 75 \times 25$

$= 1875 \text{ mm}^2$.



Area of the element 1, A_1

$$A_1 = \frac{\text{Area at node 1} + \text{Area at node 2}}{2}$$

$$= \frac{3750 + 2812.5}{2} = 3281.25 \text{ mm}^2.$$

Area of the element 2, A_2

$$A_2 = \frac{\text{Area at node 2} + \text{Area at node 3}}{2}$$

$$= \frac{2812.5 + 1875}{2} = 2343.75 \text{ mm}^2.$$

Body force Factor

$$\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = \left\{ \frac{\rho_1 A_1 l_1}{2} \right\} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$$

$$= \left\{ \frac{0.82 \times 10^{-4} \times 3281.25 \times 300}{2} \right\} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$$

$$= 40.359 \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$$

$$\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = \begin{Bmatrix} 40.359 \\ 40.359 \end{Bmatrix}$$

$$\begin{Bmatrix} F_2 \\ F_3 \end{Bmatrix} = \left\{ \frac{\rho_2 A_2 l_2}{2} \right\} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$$

$$= \left\{ \frac{0.82 \times 10^{-4} \times 2343.75 \times 300}{2} \right\} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$$

$$= 28.828 \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$$

$$= \begin{Bmatrix} 28.828 \\ 28.828 \end{Bmatrix}$$

Assemble

$$\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix} = \begin{Bmatrix} 40.359 \\ 40.359 + 28.828 + 100 \\ \cancel{40.359} \; 28.828 \end{Bmatrix}$$

$$= \begin{Bmatrix} 40.359 \\ 169.187 \\ 28.828 \end{Bmatrix}$$

For element 1,

$$\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = \frac{A_1 E}{l} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

$$= \frac{3281.25 \times 2 \times 10^{5}}{300} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

$$= 2.187 \times 10^{6} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

$$= 1 \times 10^{6} \begin{array}{cc} a_{11} & a_{12} \\ \end{array} \begin{bmatrix} 2.187 & -2.187 \\ -2.187 & 2.187 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

(a_{21}, a_{22})

For element 2,

$$\begin{Bmatrix} F_2 \\ F_3 \end{Bmatrix} = \frac{A_2 E}{l_2} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} u_2 \\ u_3 \end{bmatrix}$$

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$$\begin{Bmatrix} F_2 \\ F_3 \end{Bmatrix} = \frac{2343.75 \times 2 \times 10^5}{300} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix}$$

$$= 1.562 \times 10^6 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix}$$

$$= 1 \times 10^6 \begin{bmatrix} \overset{a_{22}}{1.562} & \overset{a_{23}}{-1.562} \\ \overset{a_{32}}{-1.562} & \overset{a_{33}}{1.562} \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix}$$

Assembling

$$\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix} = 1 \times 10^6 \begin{bmatrix} 2.187 & -2.187 & 0 \\ -2.187 & 3.749 & -1.562 \\ 0 & -1.562 & 1.562 \end{bmatrix} \begin{Bmatrix} 0 \\ u_2 \\ u_3 \end{Bmatrix}$$

$$\begin{Bmatrix} \overset{169.187}{F_2} \\ \overset{28.828}{F_3} \end{Bmatrix} = 1 \times 10^6 \begin{bmatrix} 3.749 & -1.562 \\ -1.562 & 1.562 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix}$$

$$1 \times 10^6 \left[3.749\, u_2 - 1.562\, u_3 \right] = 169.187$$

$$1 \times 10^6 \left[-1.562\, u_2 + 1.562\, u_3 \right] = 28.828$$

$$2.187 \times 10^6 u_2 = 198.015$$

$$u_2 = 9.054 \times 10^{-5} \text{ mm}$$

$$1 \times 10^6 \left[3.749 (9.054 \times 10^{-5}) - 1.562\, u_3 \right] = 169.87$$

$$339.441 - 1.562 \times 10^6 u_3 = 169.87$$

$$1.562 \times 10^6 u_3 = 169.571$$

$$u_3 = 1.085 \times 10^{-4} \text{ mm}$$

$$= 10.85 \times 10^{-5} \text{ mm}$$

$$\sigma_1 = E_1 \left[\frac{du}{dx}\right]$$

$$= 2\times10^5 \frac{(u_2-u_1)}{300}$$

$$\sigma_1 = 0.060 \; N/mm^2$$

$$\sigma_2 = E_1 \frac{du}{dx}$$

$$= 2\times10^5 \frac{(u_3-u_2)}{300}$$

$$= 0.0119 \; N/mm^2 \quad (0.012 \; N/mm^2)$$

$$\{R\} = \{K\}\{u^*\} - \{F\}$$

$$= 1\times10^6 \begin{bmatrix} 2.187 & -2.187 & 0 \\ -2.187 & 3.749 & -1.562 \\ 0 & -1.562 & 1.562 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} - \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix}$$

$$= 1\times10^6 \begin{bmatrix} 2.187 & -2.187 & 0 \\ -2.187 & 3.749 & -1.562 \\ 0 & -1.562 & 1.562 \end{bmatrix} \begin{Bmatrix} 0 \\ 9.054\times10^{-5} \\ 10.85\times10^{-5} \end{Bmatrix} - \begin{Bmatrix} 40.359 \\ 169.187 \\ 28.828 \end{Bmatrix}$$

$$= 1\times10^6 \begin{bmatrix} \cancel{3.749} & -1.562 \\ -1.562 & \cancel{1.562} \end{bmatrix} \begin{Bmatrix} \cancel{9.054\times10^{-5}} \\ \cancel{10.85\times10^{-5}} \end{Bmatrix} - \begin{Bmatrix} \cancel{169.187} \\ \cancel{28.828} \end{Bmatrix}$$

$$= 1\times10^6 \begin{bmatrix} -1.98\times10^{-4} \\ 1.699\times10^{-4} \\ 2.805\times10^{-5} \end{bmatrix} -$$

$$= \begin{Bmatrix} 1.98\times10^{2} \\ 1.699\times10^{2} \\ 2.805\times10^{1} \end{Bmatrix} - \begin{Bmatrix} 40.359 \\ 169.187 \\ 28.828 \end{Bmatrix}$$

$$= \begin{Bmatrix} -238.359 \\ 0.713 \\ -0.778 \end{Bmatrix}$$

$$\{R\} = \begin{bmatrix} -238.379 \\ 0 \\ 0 \end{bmatrix}$$

$$R_1 + R_2 + R_3 = -238.359 + 0 + 0$$

$$= -238.359 \text{ N.}$$

$$F_1 + F_2 + F_3 = 40.359 + 169.187 + 28.82$$

$$= 238.3 \text{ N.}$$

Hence verified.