TRUSSES

A TRUSS is defined as a structure made up of serveral bars, rivetted (or) welded together.

The following assumptions are made while finding the forces in a Truss

(1) all the members are pin jointed

(2) The truss is loaded only at the joints

(3) The self weight of the members are neglected unless stated.

Applications:-

1. Roof Truss,
2. Ware house,
3. Railway platform,
4. Towers,
5. crane truss
6. Bridge truss
7. Sport stadium Truss.

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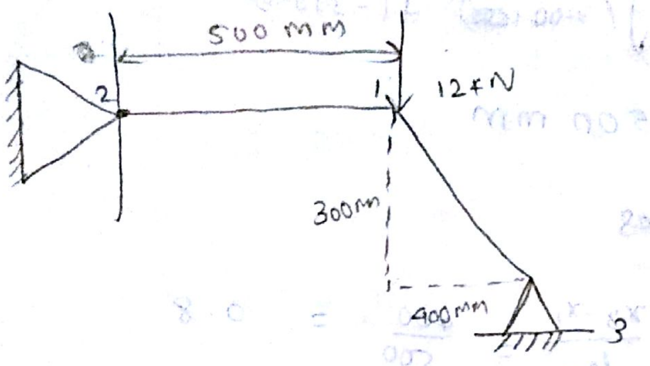
Q) For the two bar truss shown in fig. Determine the displacements of node 1 and the stress in element 2 (once 1, 3)



youngs modulus $E = 700$ GPa, Area $= 200\ mm^2$.

Sol:

The co-ordinates of various nodes are

node 1 = (0, 0)

node 2 = (−500, 0)

node 3 = (400, −300)

For element ①.

$$le_1 = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(-500-0)^2 + (0-0)^2}$$

$$le_1 = 500\ mm.$$

Direction cosines

$$l_1 = \frac{x_2 - x_1}{le_1} = \frac{-500}{500} = -1$$

$$m_1 = \frac{y_2 - y_1}{le_1} = 0.$$

element 2

$$le_2 = \sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2}$$

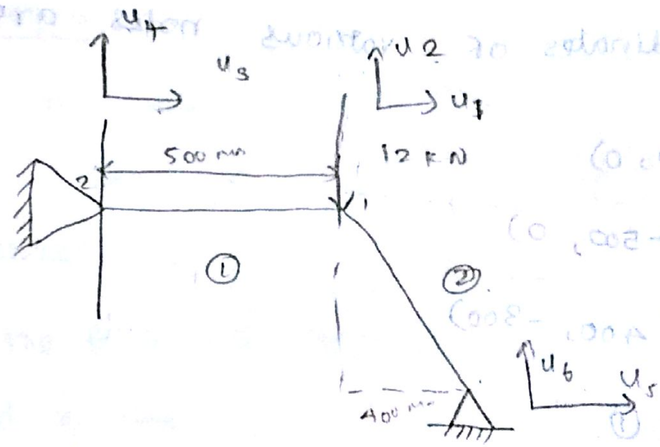
$$= \sqrt{(400 + \cancel{500}^{0})^2 + (-300 - 0)^2}$$

$$= 500 \text{ mm}$$

Direction cosines

$$l_2 = \frac{x_3 - x_1}{le_2} = \frac{400}{500} = 0.8$$

$$m_2 = \frac{y_3 - y_1}{le_2} = \frac{-300}{500} = -0.6.$$



$f_1 = 0$

$f_2 = -12 \times 10^3$

0

0

0

0

For element 1 (displacement u_1, u_2, u_3, u_4)

$$[K]_{e1} = \frac{A_1 E_1}{le_1} \begin{bmatrix} l_1^2 & l_1 m_1 & -l_1^2 & -l_1 m_1 \\ l_1 m_1 & m_1^2 & -l_1 m_1 & -m_1^2 \\ -l_1^2 & -l_1 m_1 & l_1^2 & l_1 m_1 \\ -l_1 m_1 & -m_1^2 & l_1 m_1 & m_1^2 \end{bmatrix}$$

$$= \frac{200 \times 70 \times 10^3}{500} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array}$$

$$= 28 \times 10^3 \, [\ \ ]_{4 \times 4}$$

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For element ② (displacement u_2, u_1, u_5 and u_6).

$$[K]_{e_1} = \frac{200\times70\times10^3}{500}\begin{array}{c}\begin{array}{cccc}1 & 2 & 5 & 6\end{array}\\ \begin{bmatrix}0.64 & -0.48 & -0.64 & 0.48\\ -0.48 & 0.36 & 0.48 & -0.36\\ -0.64 & 0.48 & 0.64 & -0.48\\ 0.48 & -0.36 & -0.48 & 0.36\end{bmatrix}\end{array}\begin{array}{c}1\\2\\5\\6\end{array}$$

$$= 28\times10^3\,[u]$$

$$[K]_{e_1} = \frac{A_2E_2}{l_{e_2}}\begin{bmatrix}l_2^2 & l_2m_2 & -l_2^2 & -l_2m_2\\ l_2m_2 & m_2^2 & -l_2m_2 & -m_2^2\\ -l_2^2 & -l_2m_2 & l_2^2 & l_2m_2\\ -l_2m_2 & -m_2^2 & +l_2m_2 & m_2^2\end{bmatrix}.$$

Assemble the stiffness matrix K

$k = 28\times10^3$

	1	2	3	4	5	6	
	1.64	-0.48	-1	0	-0.64	0.48	1
	-0.48	0.36	0	0	0.48	-0.36	2
	-1	0	1	0	0	0	3
	0	0	0	0	0	0	4
	-0.64	0.48	0	0	0.64	-0.48	5
	0.48	-0.36	0	0	-0.48	0.36	6

Finite element equation,

$$\{F\} = [K]\times\{u\}$$

$$\begin{Bmatrix}F_1\\F_2\\F_3\\F_4\\F_5\\F_6\end{Bmatrix} = 28\times10^3\begin{bmatrix}1.64 & -0.48 & -1 & 0 & -0.64 & 0.48\\ -0.48 & 0.36 & 0 & 0 & 0.48 & -0.36\\ -1 & 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ -0.64 & 0.48 & 0 & 0 & 0.64 & -0.48\\ 0.48 & -0.36 & 0 & 0 & -0.48 & 0.36\end{bmatrix}\begin{Bmatrix}u_1\\u_2\\u_3\\u_4\\u_5\\u_6\end{Bmatrix}$$

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Apply boundry conditions,

$$u_3 = u_4 = u_5 = u_6 = 0, \qquad F_2 = -12 \times 10^3 \text{ N}.$$

$$\begin{Bmatrix} 0 \\ -12\times10^3 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} = 28\times10^3 \begin{bmatrix} 1.64 & -0.48 & -1 & 0 & -0.64 & 0.48 \\ -0.48 & 0.36 & 0 & 0 & 0.48 & -0.36 \\ -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -0.64 & 0.48 & 0 & 0 & 0.64 & -0.48 \\ 0.48 & -0.36 & 0 & 0 & -0.48 & 0.36 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

$$\begin{Bmatrix} 0 \\ -12\times10^3 \end{Bmatrix} = 28\times10^3 \begin{bmatrix} 1.64 & -0.48 \\ -0.48 & 0.36 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

$$28\times10^3 (1.64\, u_1 - 0.48\, u_2) = 0$$

$$28\times10^3 (-0.48\, u_1 + 0.36\, u_2) = -12\times10^3.$$

$$28\times10^3 (0.7872\, u_1 - 0.2304\, u_2) = 0$$

$$28\times10^3 (-0.7872\, u_1 + 0.5904) = -1.968\times10^3$$

$$28\times10^3 \times 0.36\, u_2 = -1.968\times10^3$$

$$u_2 = -1.952 \text{ mm}$$

$$28\times10^3 (1.64\, u_1 - 0.48(-1.952)) = 0.$$

$$28\times10^3 \times 1.64\, u_1 = -26240$$

$$u_1 = -0.571 \text{ mm.}$$

Stress,

$$\sigma_1 = \frac{E_1}{l_{e_1}} \begin{bmatrix} -l_1 & -m_1 & l_1 & m_1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix}$$

$$\sigma_1 = \frac{70 \times 10^3}{500} \begin{bmatrix} 1 & 0 & -1 & 0 \end{bmatrix} \begin{Bmatrix} -0.571 \\ -1.952 \\ 0 \\ 0 \end{Bmatrix}$$

$$= 140 \begin{bmatrix} -0.571 \end{bmatrix}$$

$$= -79.94 \ N/m^2$$

$$\sigma_2 = \frac{70 \times 10^3}{500} \begin{bmatrix} -0.8 & +0.6 & 0.8 & -0.6 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_5 \\ u_6 \end{Bmatrix}$$

$$= 140 \begin{bmatrix} -0.8 & +0.6 & 0.8 & -0.6 \end{bmatrix} \begin{Bmatrix} -0.571 \\ -1.952 \\ 0 \\ 0 \end{Bmatrix}$$

$$= 140 \begin{bmatrix} 0.456 - 1.1712 \end{bmatrix}$$

$$\sigma_2 = -100 \ N/mm^2$$