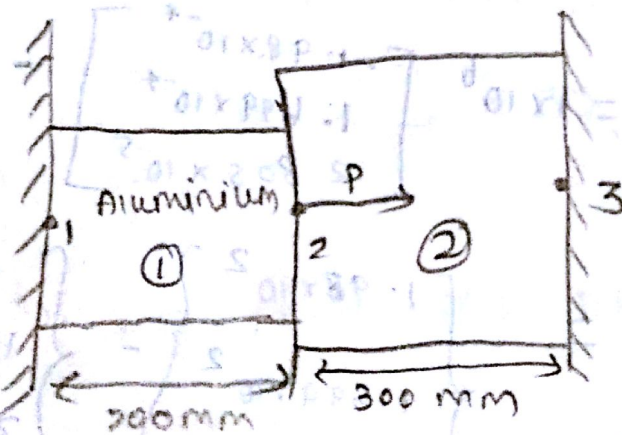


Q. An axial load of $4 \times 10^5 \text{ N}$ is applied at 30°C to the rod as shown in fig. The temperature is then raised to 60°C .

Calculate the following:

- (i) Assemble the k and F matrices,
- (ii) Nodal displacements,
- (iii) Stresses in each element,
- (iv) Reactions at each nodal point.



For aluminium

$$A_1 = 1000 \text{ mm}^2$$

$$E_1 = 0.7 \times 10^5 \text{ N/mm}^2$$

$$\alpha_1 = 23 \times 10^{-6} / ^\circ \text{C}$$

For steel

$$A_2 = 1500 \text{ mm}^2$$

$$E_2 = 2 \times 10^5 \text{ N/mm}^2$$

$$\alpha_2 = 12 \times 10^{-6} / ^\circ \text{C}$$

Given data :-

$$P = 4 \times 10^5 \text{ N}, \quad \Delta T = 60 - 30 = 30^\circ \text{C}$$

$$l_1 = 200 \text{ mm}, \quad l_2 = 300 \text{ mm}.$$

To find :-

(i) Assemble the $[K]$ and $\{F\}$ matrices

(ii) u_1, u_2, u_3

(iii) σ_1, σ_2

(iv) R_1, R_2, R_3 .

Sol:-

Finite element equation

$$\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = \frac{A_1 E_1}{l_1} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

For element 1: Nodes (1, 2)

$$\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = \frac{A_1 E_1}{l_1} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

$$= \frac{1000 \times 0.7 \times 10^5}{200} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

$$= 1 \times 10^5 \begin{bmatrix} 3.5 & -3.5 \\ -3.5 & 3.5 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

For element 2: Nodes (2, 3)

$$\begin{Bmatrix} F_2 \\ F_3 \end{Bmatrix} = \frac{A_2 E_2}{l_2} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix}$$

$$= 1 \times 10^5 \begin{bmatrix} 10 & -10 \\ -10 & 10 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix}$$

Assembling equation,

$$\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix} = 1 \times 10^5 \begin{bmatrix} 3.5 & -3.5 & 0 \\ -3.5 & 13.5 & -10 \\ 0 & -10 & 10 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} \quad \text{--- (i)}$$

Assembling of $\{F\}$ matrix

$$\{F\} = EA \alpha \Delta T \begin{Bmatrix} -1 \\ 1 \end{Bmatrix}$$

For element (1) $\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = E_1 A_1 \alpha_1 \Delta T \begin{Bmatrix} -1 \\ 1 \end{Bmatrix}$

$$= 0.7 \times 10^5 \times 1000 \times 23 \times 10^{-6} \times 30 \begin{Bmatrix} -1 \\ 1 \end{Bmatrix}$$

$$= 1 \times 10^5 \begin{Bmatrix} -0.483 \\ 0.483 \end{Bmatrix}$$

For element (2) $\begin{Bmatrix} F_2 \\ F_3 \end{Bmatrix} = E_2 A_2 \alpha_2 \Delta T \begin{Bmatrix} -1 \\ 1 \end{Bmatrix}$

$$= 1 \times 10^5 \begin{Bmatrix} -1.08 \\ 1.08 \end{Bmatrix}$$

Assembling

$$\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix} = 1 \times 10^5 \begin{Bmatrix} -0.483 \\ -0.483 - 1.08 + 1 \\ 1.08 \end{Bmatrix} = 1 \times 10^5 \begin{Bmatrix} -0.483 \\ 3.48 \\ 1.08 \end{Bmatrix}$$

Applying boundary conditions in (i)

$$u_1 = 0, \quad u_2, \quad u_3 = 0$$

$$1 \times 10^5 \begin{Bmatrix} -0.483 \\ 3.403 \\ 1.08 \end{Bmatrix} = 1 \times 10^5 \begin{bmatrix} 3.5 & -3.5 & 0 \\ -3.5 & 13.5 & -10 \\ 0 & -10 & 0 \end{bmatrix} \begin{Bmatrix} 0 \\ u_2 \\ 0 \end{Bmatrix}$$

$$13.5 u_2 = 1 \times 10^5 (3.403)$$

$$u_2 = 0.5251 \text{ mm},$$

Thermal stress,

$$\sigma = E \frac{\Delta u}{\Delta x} = E \alpha \Delta T$$

$$\sigma_1 = E_1 \frac{(u_2 - u_1)}{\Delta x_1} = E_1 \alpha_1 \Delta T$$

$$= \frac{0.75 \times 10^5 (0.5251 - 0)}{200} = 0.75 \times 10^5 \times 2.3 \times 10^{-6} \times 30$$

$$\sigma_1 = 39.935 \text{ N/mm}^2 \text{ (Tensile)}$$

$$\sigma_2 = E_2 \frac{(u_3 - u_2)}{\Delta x_2} = E_2 \alpha_2 \Delta T$$

$$\sigma_2 = -240.066 \text{ N/mm}^2 \text{ [compressive]}$$

To find (c) calculate the reaction force.

$$\{R\} = [K] \{u\} - \{F\}$$

$$\begin{Bmatrix} R_1 \\ R_2 \\ R_3 \end{Bmatrix} = 1 \times 10^5 \begin{bmatrix} 3.5 & -3.5 & 0 \\ -3.5 & 13.5 & -10 \\ 0 & -10 & 10 \end{bmatrix} \begin{Bmatrix} 0 \\ 0.5251 \\ 0 \end{Bmatrix} - 10^5 \begin{Bmatrix} -0.483 \\ 3.403 \\ 1.08 \end{Bmatrix}$$

$$\begin{Bmatrix} R_1 \\ R_2 \\ R_3 \end{Bmatrix} = 1 \times 10^5 \begin{Bmatrix} -0.8223 \\ 3.905 \\ -2.521 \end{Bmatrix} - 1 \times 10^5 \begin{Bmatrix} -0.483 \\ 3.403 \\ 1.08 \end{Bmatrix}$$

$$= 1 \times 10^5 \begin{Bmatrix} -0.3393 \\ 0 \\ -3.601 \end{Bmatrix}$$

$$R_1 = -0.3993 \times 10^5 \text{ N}$$

$$R_2 = 0$$

$$R_3 = -3.601 \times 10^5 \text{ N}$$

verification,

$$R_1 + R_2 + R_3 = -0.3993 \times 10^5 + 0 - 3.601 \times 10^5$$

$$= -4 \times 10^5 \text{ N (APPLIED force)}$$

Hence its verified.