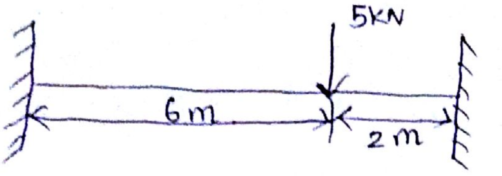
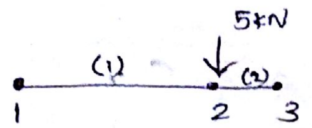
① Calculate the deflection under the load in the statically indeterminate beam in figure 1., and predict the shear force and bending moment distributions.



$E = 200 \times 10^3$ M/pa

$I = 4 \times 10^{-6} m^4$.

The finite element model for the beam need only consist of two elements.



Stiffness matrix for element (1)

$$K^{(1)} = \frac{EI}{L^3} \begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

$$= \frac{200 \times 10^3 \times 10^6 \times 4 \times 10^{-6}}{(6)^3} \begin{bmatrix} 12 & 6(6) & -12 & 6(6) \\ 6(6) & 4(6)^2 & -6(6) & 2(6)^2 \\ -12 & -6(6) & 12 & -6(6) \\ 6(6) & 2(6)^2 & -6(6) & 4(6)^2 \end{bmatrix}$$

$$= \frac{0.8 \times 10^6}{216} \begin{bmatrix} 12 & 36 & -12 & 36 \\ 36 & 144 & -36 & 72 \\ -12 & -36 & 12 & -36 \\ 36 & 72 & -36 & 144 \end{bmatrix}$$

$$= 10^6 \begin{bmatrix} 0.044 & 0.133 & -0.044 & 0.133 \\ 0.133 & 0.533 & -0.133 & 0.266 \\ -0.044 & -0.133 & 0.044 & -0.133 \\ 0.133 & 0.266 & -0.133 & 0.533 \end{bmatrix}$$

Stiffness matrix for element (2)

$$[K]^{(2)} = \frac{0.8\times10^6}{(2)^3}\begin{bmatrix} 12 & 6(2) & -12 & 6(2) \\ 6(2) & 4(2)^2 & -6(2) & 2(2)^2 \\ -12 & -6(2) & 12 & -6(2) \\ 6(2) & 2(2)^2 & -6(2) & 4(2)^2 \end{bmatrix}$$

$$= 10^6\begin{bmatrix} 1.2 & 1.2 & -1.2 & 1.2 \\ 1.2 & 1.6 & -1.2 & 0.8 \\ -1.2 & -1.2 & 1.2 & -1.2 \\ 1.2 & 0.8 & -1.2 & 1.6 \end{bmatrix}$$

Including the load of 5kN applied at node 2, the final set of system sequation is

$$10^6\begin{bmatrix} 0.044 & 0.133 & -0.044 & 0.133 & 0 & 0 \\ 0.133 & 0.533 & -0.133 & 0.266 & 0 & 0 \\ -0.044 & -0.133 & 1.244 & 1.067 & -1.2 & 1.2 \\ 0.133 & 0.266 & 1.067 & 2.133 & -1.2 & 0.8 \\ 0 & 0 & -1.2 & -1.2 & 1.2 & -1.2 \\ 0 & 0 & 1.2 & 0.8 & -1.2 & 1.6 \end{bmatrix}\begin{bmatrix} v_1 \to 0 \\ \theta_1 \to 0 \\ v_2 \\ \theta_2 \\ v_3 \to 0 \\ \theta_3 \to 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -5\times10^3 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$10^6\begin{bmatrix} 1.244 & 1.067 \\ 1.067 & 2.133 \end{bmatrix}\begin{bmatrix} v_2 \\ \theta_2 \end{bmatrix} = \begin{Bmatrix} -5\times10^3 \\ 0 \end{Bmatrix}$$

$$1.244\times10^6\, v_2 + 1.067\times10^6\, \theta_2 = -5\times10^3 \rightarrow (1)$$

$$1.067\times10^6\, v_2 + 2.133\times10^6\, \theta_2 = 0 \rightarrow (2)$$

Solving the equations (1) & (2)

$$v_2 = 0.007037 \text{ m},$$

$$\theta_2 = 0.00352 \text{ rad}$$