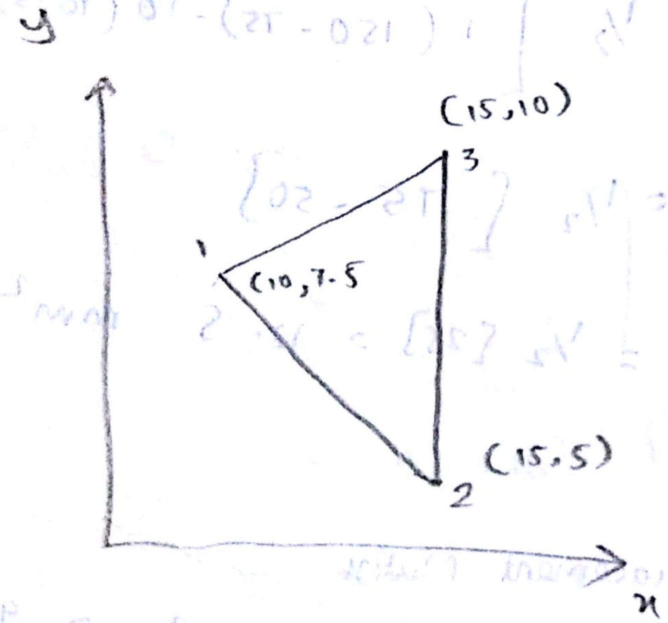
Calculate the element stresses σ_x, σ_y, τ_{xy}, σ_1 and σ_2 and the Principle angle θ_p for the element as shown.



The nodal displacements are $u_1 = 2$ mm, $v_1 = 1$ mm, $u_2 = 0.5$ mm, $v_2 = 0$ mm, $u_3 = 3$ mm, $v_3 = 1$ mm take $E = 2.1 \times 10^5$ N/mm^2 and $P = 0.25$. assume

Plane stress condition.

To find :-

σ_x - normal stress

σ_y - normal stress

τ_{xy} - shear stress

σ_1 - maximum normal stress

σ_2 - minimum "

θ_p — Principle angle.

Sol:-

$$Area = \frac{1}{2}\begin{vmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} 1 & 10 & 7.5 \\ 1 & 15 & 5 \\ 1 & 15 & 10 \end{vmatrix}$$

$$= \frac{1}{2}\left[1(150-75)-10(10-5)+7.5(15-15)\right]$$

$$= \frac{1}{2}\left[75-50\right]$$

$$Area = \frac{1}{2}[25] = 12.5 \ mm^2$$

Stress displacement Matrix

$$[B] = \frac{1}{2A}\begin{bmatrix} q_1 & 0 & q_2 & 0 & q_3 & 0 \\ 0 & r_1 & 0 & r_2 & 0 & r_3 \\ r_1 & q_1 & r_2 & q_2 & r_3 & q_3 \end{bmatrix}$$

$$q_1 = (y_2 - y_3) = (5-10) = -5$$

$$q_2 = (y_3 - y_1) = (10 - 7.5) = 2.5$$

$$q_3 = (y_1 - y_2) = (7.5 - 5) = 2.5$$

$$r_1 = (x_3 - x_2) = (15 - 15) = 0$$

$$r_2 = (x_1 - x_3) = (10 - 15) = -5$$

$$r_3 = (x_2 - x_1) = (15 - 10) = +5$$

$$[B] = \frac{1}{2 \times 12.5} \begin{bmatrix} -5 & 0 & 2.5 & 0 & 2.5 & 0 \\ 0 & 0 & 0 & -5 & 0 & +5 \\ 0 & -5 & -5 & 2.5 & +5 & 2.5 \end{bmatrix}$$

$$= 0.04 \times 2.5 \begin{bmatrix} -2 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & -2 & 0 & +2 \\ 0 & -2 & -2 & 1 & 2 & 1 \end{bmatrix}$$

$$= 0.1$$

$$[D] = \frac{E}{1 - v^2} \begin{bmatrix} 1 & v & 0 \\ v & 1 & 0 \\ 0 & 0 & \frac{1-v}{2} \end{bmatrix}$$

$$= \frac{2.1 \times 10^5}{1 - (0.25)^2} \begin{bmatrix} 1 & 0.25 & 0 \\ 0.25 & 1 & 0 \\ 0 & 0 & \frac{1-0.25}{2} \end{bmatrix}$$

$$= 56 \times 10^3 \begin{bmatrix} 4 & 1 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 1.5 \end{bmatrix}$$

$$[D]\{B\} = 56\times10^{3}\begin{bmatrix}4 & 1 & 0\\ 1 & 4 & 0\\ 0 & 0 & 1.5\end{bmatrix} \times 0.1\begin{bmatrix}-2 & 0 & 1 & 0 & 1 & 0\\ 0 & 0 & 0 & -2 & 0 & 2\\ 0 & -2 & -2 & 1 & 2 & 1\end{bmatrix}$$

$$= 5.6\times10^{3}\begin{bmatrix}-8+0+0 & 0+0+0 & 4+0+0 & 0-2+0 & 4+0+0 & 0+2\ldots\\ -2+0+0 & 0+0+0 & 1+0+0 & 0-8+0 & 1+0+0 & 0+8\ldots\\ 0+0+0 & 0+0-3 & 0+0-3 & 0+0+1.5 & 0+0+3 & 0\ldots\end{bmatrix}$$

$$= 5.6\times10^{3}\begin{bmatrix}-8 & 0 & 4 & -2 & 4 & 2\\ -2 & 0 & 1 & -8 & 1 & 8\\ 0 & -3 & -3 & 1.5 & 3 & 1.5\end{bmatrix}$$

Stress $\sigma = [B]\times[D] \times \{u\}$

$$= 5.6\times10^{3}\begin{bmatrix}-8 & 0 & 4 & -2 & 4 & 2\\ -2 & 0 & 1 & -8 & 1 & 8\\ 0 & -3 & -3 & 1.5 & 3 & 1.5\end{bmatrix}\begin{Bmatrix}u_1\\ v_1\\ u_2\\ v_2\\ u_3\\ v_3\end{Bmatrix} = \begin{Bmatrix}2\\ 1\\ 0.5\\ 0\\ 3\\ 1\end{Bmatrix}$$

$$= 5.6\times10^{3}\begin{bmatrix}-16+0+2+0+12+2\\ -4+0+0.5-0+3+8\\ 0-3-1.5+0+9+1.5\end{bmatrix}$$

$$\sigma = 5.6\times10^{3}\begin{bmatrix}0\\ 7.5\\ 6\end{bmatrix}$$

$$\sigma = \begin{bmatrix}0\\ 42000\\ 33600\end{bmatrix} = \begin{bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{bmatrix}$$

$\sigma_x = 0$

$\sigma_y = 4.2\times10^{3}$ N/m²

$\tau_{xy} = 33.6\times10^{3}$ N/m²

$$\sigma_{max} = \frac{\sigma_x+\sigma_y}{2} + \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{max} = \frac{0 + 4.2\times10^3}{2} + \sqrt{\left(\frac{0 - 4.2\times10^3}{2}\right)^2 + (33.6\times10^3)^2}$$

$$= 2.1\times10^3 + 39622.72$$

$$\sigma_{max} = 60.622\times10^3 \ N/mm^2$$

Minimum nominal stress, (σ_{min})

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2_{xy}}$$

$$= 2.1\times10^3 - 39622.72$$

$$= -18.622\times10^3 \ N/mm^2$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

$$\tan 2\theta_p = \frac{2\times33.6\times10^3}{0 - 42\times10^3}$$

$$\tan 2\theta_p = -1.6$$

$$2\theta_p = \tan^{-1}(-1.6)$$

$$2\theta_p = -57.99$$

$$\theta_p = -29^\circ$$

Note :-

For calculating strain, $e = [B]\times\{u\}$