16ME401 Finite Element Analysis

1．Distinguish between CST and LST triangular element．

| CST | LST |
| ---: | ---: |
| 1．Constant strain triangular element | 1．Linear strain triangular element |
| 2．It has 6 nodal displacement | 2．It has 12 nodal displacement |
| 3．It has 3 shape function | 3．It has 6 shape function |
| 4．It is suitable for large elements | 4．It is suitable for small elements |

2．What meant by plane stress analysis？
Plane stress is defined to be a state of stress in which the normal stress and shear stress directed perpendicular to the plane are assumed to be zero．
3．Write a strain－displacement matrix for CST element．

$$
[B]=\frac{1}{2 A}\left[\begin{array}{cccccc}
p_{1} & 0 & p_{2} & 0 & p_{3} & 0 \\
0 & r_{1} & 0 & r_{2} & 0 & r_{3} \\
r_{1} & p_{1} & r_{2} & p_{2} & r_{3} & p_{3}
\end{array}\right] \quad \mathrm{A}=\text { Area of the element }
$$

$$
p_{1}=y_{2}-y_{3} ; p_{2}=y_{3}-y_{1} ; p_{3}=y_{1}-y_{2} \quad r_{1}=x_{3}-x_{2} ; r_{2}=x_{1}-x_{3} ; r_{3}=x_{2}-x_{1}
$$

4．Give the strain－displacement matrix equation for an axisymmetric triangular element．

$$
[B]=\frac{1}{2 A}\left[\begin{array}{ccccccc}
\beta_{1} & 0 & \beta_{2} & 0 & \beta_{3} & 0 \\
\frac{\alpha_{1}}{r}+\beta_{1}+\frac{\gamma_{1} z}{r} & 0 & \frac{\alpha_{2}}{r}+\beta_{2}+\frac{\gamma_{2} z}{r} & 0 & \frac{\alpha_{3}}{r}+\beta_{3}+\frac{\gamma_{3} z}{r} & 0 \\
0 & \gamma_{1} & 0 & \gamma_{2} & 0 & \gamma_{3} \\
\gamma_{1} & \beta_{1} & \gamma_{2} & \beta_{2} & \gamma_{3} & \beta_{3}
\end{array}\right]
$$

5．Specify the Mechanical components related with axisymmetric concept．
Pressure vessels，cylinders loaded by uniform internal or external pressures，flywheel， turbine discs，and circular footings resting on a soil mass
6．．Define plane strain analysis．
Plane strain is defined to be state of strain normal to the xy plane and the shear strains are assumed to be zero
7．Give the stress－strain constitutive relation for a linear，elastic，isotropic material under plane－strain condition．

$$
\left\{\begin{array}{l}
\sigma_{x} \\
\sigma_{y} \\
\tau_{x y}
\end{array}\right\}=\frac{E}{(1+\mu)(1-2 \mu)}\left[\begin{array}{ccc}
1-\mu & \mu & 0 \\
\mu & 1-\mu & 0 \\
0 & 0 & \frac{1-2 \mu}{2}
\end{array}\right]\left\{\begin{array}{c}
\varepsilon_{x} \\
\varepsilon_{y} \\
\gamma_{x y}
\end{array}\right\}
$$

8．What are the ways in which a three dimensional problem can be reduced to a two dimensional approach？

1．Plane stress：one dimensional is too small when compared to other two dimensions．［ example：gear－thickness is small］

2．Plane strain：one dimensional is too large when compared to other two dimensions．［ example：long pipe－length is long compared to diameter］

3．Axisymmetric：Geometry is symmetric about the axis［ example：cooling tower］
9. Write down the governing differential equation for the steady state heat transfer equation.
The governing differential equation for the steady state one-dimensional conduction heat transfer with convective heat loss from lateral surfaces is given by

$$
k \frac{d^{2} T}{d x^{2}}+q=\left(\frac{P}{A_{c}}\right) h\left(T-T_{\infty}\right)
$$

where
$k=$ coefficient of thermal conductivity of the material,
$T=$ temperature ,
$q=$ internal heat source per unit volume,
$P=$ perimeter,
$A_{c}=$ the cross-sectional area,
$h=$ convective heat transfer coefficient, and
$T_{\infty}=$ ambient temperature.
10. Write down the finite element analysis equation for one dimensional element with conduction, surface convection and internal heat generation.
$\left(\frac{k A}{l}\left[\begin{array}{cc}1 & -1 \\ -1 & 1\end{array}\right]+\frac{h p l}{6}\left[\begin{array}{ll}2 & 1 \\ 1 & 2\end{array}\right]+h A\left[\begin{array}{ll}0 & 0 \\ 0 & 1\end{array}\right]\right)\left\{\begin{array}{l}T_{i} \\ T_{j}\end{array}\right\}=\left[\frac{h T_{\infty} P l}{2}\left[\begin{array}{l}1 \\ 1\end{array}\right]+h T_{\infty} A\left[\begin{array}{l}0 \\ 1\end{array}\right]+\frac{Q A l}{2}\left[\begin{array}{l}1 \\ 1\end{array}\right]\right]$
11.List out the equivalent nodal loads for typical loading on beams.
(i)

(ii)

(iii)


12. Why higher order elements are needed?

The higher order elements can capture variations in stress in an element, such as the stress occurring near fillets, holes, etc., thus providing accuracy.
13. How do you attempt to FEA model this?

$\Rightarrow$


14. What are the classifications of coordinates?

Global coordinates
Local coordinates
Natural coordinates
15. Write down the stress-strain relationship matrix for an axisymmetric triangular element.

$$
[D]=\frac{E}{(1+\mu)(1-2 \mu)}\left[\begin{array}{cccc}
(1-\mu) & \mu & \mu & 0 \\
\mu & (1-\mu) & \mu & 0 \\
\mu & \mu & (1-\mu) & 0 \\
0 & 0 & 0 & \left(\frac{1-2 \mu}{2}\right)
\end{array}\right]
$$

16. The plane wall shown in figure is 1 m thick. The left surface of the wall is maintained at a constant temperature of $200^{\circ} \mathrm{C}$, and the right surface is insulated. How do you FEA model attempt to determine the temperature distribution through the wall thickness?


Figure 1. conduction in a plane wall subjected to uniform heat generation.

Solution:


Figure Discretized FEA model
This problem is assumed to be approximated as a one-dimensional heat transfer problem. The discretized model of wall is shown in figure. For simplicity, we use four equal-length elements all with unit cross-sectional area $\left(A=1 m^{2}\right)$. The unit area represents typical cross sectional of the wall. The perimeter of the wall model is then insulated to obtain the correct conditions. Because no convection occurs, $h$ is equal to zero; therefore, there is no convection contribution to stiffness matrix $k$.

