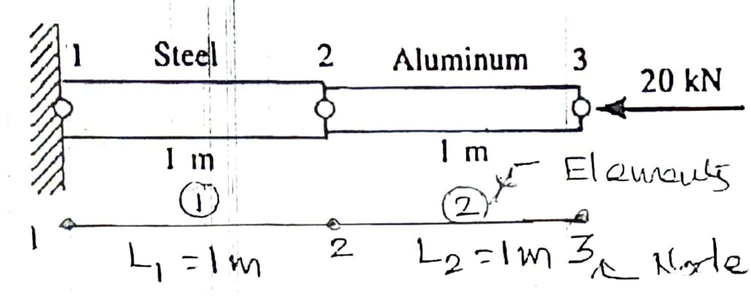
For the bar assemblage shown in Figure 0.0, determine the nodal displacements, the forces in each element, and the reactions. Use the direct stiffness method for these problems.



$E_{st} = 200$ GPa
$A_{st} = 4 \times 10^{-4}$ m^2
$E_{al} = 70$ GPa
$A_{al} = 2 \times 10^{-4}$ m^2

Stiffness matrix For Element ①

$$k^{(1)} = \frac{A_1 E_1}{L_1}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$k^{(1)} = 4\times10^{-4}(m^2)\times200\times10^{6}\left(\frac{KN}{m^2}\right)\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$= 800\times10^{2}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\frac{KN}{m}$$

$$\begin{array}{cc} 1 & 2 \end{array}$$
$$= 10^{2}\begin{bmatrix} 800 & -800 \\ -800 & 800 \end{bmatrix}\begin{array}{c} 1 \\ 2 \end{array} \frac{KN}{m}$$

Stiffness matrix For Element ②

$$k^{(2)} = \frac{A_2 E_2}{L_2}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$= \frac{2\times10^{-4}\times70\times10^{6}}{1}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$k^{(2)} = 140\times10^{2}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$\begin{array}{cc} 2 & 3 \end{array}$$
$$= 10^{2}\begin{bmatrix} 140 & -140 \\ -140 & 140 \end{bmatrix}\begin{array}{c} 2 \\ 3 \end{array} \frac{KN}{m}$$

Global matrix

$$\begin{array}{ccc} 1 & 2 & 3 \end{array}$$
$$k^{(G)} = 10^{2}\begin{bmatrix} 800 & -800 & 0 \\ -800 & 800+140 & -140 \\ 0 & -140 & 140 \end{bmatrix}\begin{array}{c} 1 \\ 2 \\ 3 \end{array} = 10^{2}\begin{bmatrix} 800 & -800 & 0 \\ -800 & 940 & -140 \\ 0 & -140 & 140 \end{bmatrix}$$

Element Equation $\Rightarrow [K][U] = \{F\}$

Global Stiff

Displacement Vector.

$$U = \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix}$$

Force Vector

$$F = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix}$$

Formulation of Element Equation.

$[K][U] = \{F\}$

$$10^2 \begin{bmatrix} 800 & -800 & 0 \\ -800 & 940 & -140 \\ 0 & -140 & 140 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix}$$

Apply Boundary Condition. $U_1 = 0,\ F_1 = 0$
$F_2 = 0,\ F_3 = -20$

$$10^2 \begin{bmatrix} \cancel{800} & \cancel{-800} & \cancel{0} \\ \cancel{-800} & 940 & -140 \\ \cancel{0} & -140 & 140 \end{bmatrix} \begin{bmatrix} \cancel{0} \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -20\,kN \end{bmatrix}$$

Apply Elimination Approach

$$10^2 \begin{bmatrix} 940 & -140 \\ -140 & 140 \end{bmatrix} \begin{bmatrix} u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 0 \\ -20\,kN \end{bmatrix}$$

generation of Simultaneous equation from Eqn

$$10^2 [940 u_2 - 140 u_3] = 0 \quad \cdots ①$$

$$u_3 = 6.741 u_2$$

$$10^2 [-140 u_2 + 140 u_3] = -20 \quad -②$$

Substituting $u_3 = 6.741 u_2 \rightarrow ②$

$$10^2 [-140 u_2 + 140 [6.74 u_2]] = -20$$

$$u_2 = -0.25 \times 10^{-3} m$$

$$u_3 = -1.678 \times 10^{-3} m \quad [-1.67857 1429 \times 10^{-3}]$$

Strain Calculation

$$\varepsilon^{(1)} = \frac{u_2 - u_1}{L_1} = \frac{-0.25 \times 10^{-3} - 0}{1} = -0.25 \times 10^{-3}$$

$$\varepsilon^{(2)} = \frac{u_3 - u_2}{L_2} = \frac{-1.678571429 \times 10^{-3} + 0.25 \times 10^{-3}}{1}$$

$$= -1.428571429 \times 10^{-3}$$

Stress Calculation [By Hooke's law]

$$\sigma^{(1)} = E_1 \varepsilon^{(1)} = 200 \times 10^6 \times -0.25 \times 10^{-3} = 50000$$

$$\sigma^{(2)} = E_2 \varepsilon^{(2)} = 70 \times 10^6 \times -1.428571429 \times 10^{-3} = 10,000$$

Theoretical stresses for this problem are easily calculated by

$$\sigma^{(1)}_{Theory} = \frac{P}{A_1} = \frac{20}{4\times10^{-4}} = 50{,}000$$

$$\sigma^{(2)}_{Theory} = P/A_2 = \frac{20}{2\times10^{-4}} = 10{,}000$$

Reaction force

$$[R] = [K][U] - [F]$$

$$\begin{bmatrix} R_1 \\ R_2 \\ R_3 \end{bmatrix} = 10^2 \begin{bmatrix} 800 & -800 & 0 \\ -800 & 940 & -140 \\ 0 & -140 & 140 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} - \begin{bmatrix} F_1 \\ F_2 \\ F_2 \end{bmatrix}$$

$R_2 \to 0$, $R_3 \to 0$, $u_1 \to 0$, $F_1 \to 0$, $F_2 \to 0$, $F_2 \to (-20)$

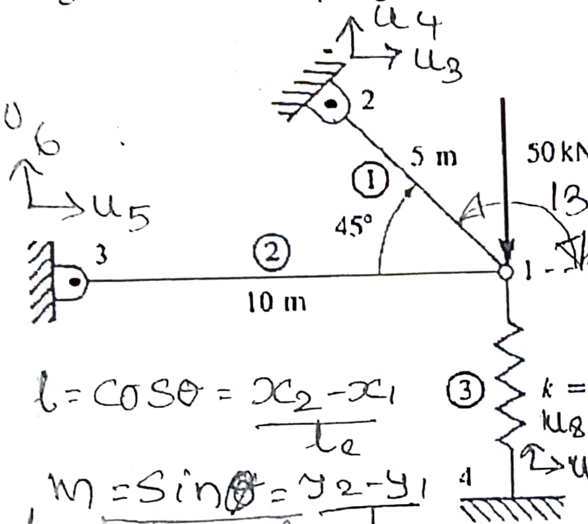
Apply BC

$$\begin{bmatrix} R_1 \\ 0 \\ 0 \end{bmatrix} = 10^2 \begin{bmatrix} 800 & -800 & 0 \\ -800 & 940 & -140 \\ 0 & -140 & 140 \end{bmatrix} \begin{bmatrix} 0 \\ u_2 \\ u_2 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ -20 \end{bmatrix}$$

$$-800\times10^2 \times u_2 - 0 = R_1$$

$$-800\times10^2 \times -1.678571429\times10^{-3} = \underline{20\ kN}$$

To illustrate how we can combine spring and bar elements in one structure, we now solve the two-bar truss supported by a spring shown in Figure .1. Both bars have E= 210 GPa and $A = 5\times10^{-4}\,m^2$. Bar one has a length of 5 m and bar two a length of 10 m. The spring stiffness is k = 2000 kN/m.



$l = \cos\theta = \dfrac{x_2 - x_1}{l_e}$

$m = \sin\theta = \dfrac{y_2 - y_1}{l_e}$

$l_e = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

Two-bar truss with spring support

Solution :

Stiffness matrix

$$K = \frac{AE}{L}\begin{bmatrix} l^2 & lm & -l^2 & -lm \\ lm & m^2 & -lm & -m^2 \\ -l^2 & -lm & l^2 & lm \\ -lm & -m^2 & lm & m^2 \end{bmatrix}$$

Element ① $\theta = 135°$

$l = \cos 135 = -707.10678\times10^{-3}$

$l^2 = 0.5$

$m = \sin 135 = +707.10678\times10^{-3}$

$m^2 = 0.5$ $[lm = -0.5]$

Stiffness matrix for Element ①

$L = 5\,m$ [node (1-2)]

$$K^{(1)} = \frac{(5\times10^{-4})\times(210\times10^{9})}{5}\begin{bmatrix} 0.5 & -0.5 & -0.5 & 0.5 \\ -0.5 & 0.5 & 0.5 & -0.5 \\ -0.5 & 0.5 & 0.5 & -0.5 \\ 0.5 & -0.5 & -0.5 & 0.5 \end{bmatrix}$$

$$= 105\times10^{5} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & -1 & -1 & 1 \\ -1 & 1 & 1 & -1 \\ -1 & 1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \end{array} \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array}$$

1/4

Element (2), $\theta = 180°$

$l^2 = 1$ $lm = 0$ $m^2 = 0$

Stiffness matrix for Element (2), node [1-5]

$L = 10\,m$

$$K^{(2)} = \frac{(5\times10^{-4})\times(210\times10^{9})}{10}\begin{bmatrix}1 & 0 & -1 & 0\\0 & 0 & 0 & 0\\-1 & 0 & 1 & 0\\0 & 0 & 0 & 0\end{bmatrix}$$

$$= 105\times10^{5}\begin{array}{c|c} \begin{array}{cccc}1 & 2 & 5 & 6\end{array} & \\ \begin{bmatrix}1 & 0 & -1 & 0\\0 & 0 & 0 & 0\\-1 & 0 & 1 & 0\\0 & 0 & 0 & 0\end{bmatrix} & \begin{array}{c}1\\2\\5\\6\end{array}\end{array}$$

Element (3), $\theta = 270°$

$l^2 = 0$, $lm = 0$ $m^2 = 1$

Stiffness matrix for Element (3) node [1-4] $= K[\ ]$

$$K^{(3)} = 20\times10^{5}\begin{array}{c|c} \begin{array}{cccc}1 & 2 & 7 & 8\end{array} & \\ \begin{bmatrix}0 & 0 & 0 & 0\\0 & 1 & 0 & -1\\0 & 0 & 0 & 0\\0 & -1 & 0 & 1\end{bmatrix} & \begin{array}{c}1\\2\\7\\8\end{array}\end{array}$$

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General Finite Element equation: $[K]\{u\} = \{F\}$

$$10^5 \begin{array}{c} \\ \\ \end{array}\begin{array}{cccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ \end{array}$$

$$10^5\begin{bmatrix} 105+105 & -105 & -105 & 105 & -105 & 0 & 0 & 0 \\ -105 & 105+105 & 105 & -105 & 0 & 0 & 0 & -20 \\ -105 & 105 & 105 & -105 & 0 & 0 & 0 & 0 \\ 105 & -105 & -105 & 105 & 0 & 0 & 0 & 0 \\ -105 & 0 & 0 & 0 & +105 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -20 & 0 & 0 & 0 & 0 & 0 & 20 \end{bmatrix} \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \end{array} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \to 0 \\ u_4 \to 0 \\ u_5 \to 0 \\ u_6 \to 0 \\ u_7 \to 0 \\ u_8 \to 0 \end{bmatrix} = \begin{bmatrix} F_1(0) \\ F_2(-50) \\ F_3(0) \\ F_4(0) \\ F_5(0) \\ F_6(0) \\ F_7(0) \\ F_8(0) \end{bmatrix} \times 10^3$$

Boundary conditions are

$u_3 = u_4 = u_5 = u_6 = u_7 = u_8 = 0$

The final matrix

$$10^5 \begin{bmatrix} 210 & -105 \\ -105 & 125 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ -50 \end{Bmatrix} \times 10^3$$

$$210 \times 10^5 u_1 - 105 \times 10^5 u_2 = 0 \quad \text{—(1)}$$

$$-105 \times 10^5 u_1 + 125 \times 10^5 u_2 = -50 \quad \text{—(2)}$$

$u_1 = -3.448 \times 10^{-3}\, m$

$u_2 = -6.896 \times 10^{-3}\, m$

Scanned by CamScanner

Stress : Stress of element (1)

$$\sigma^{(1)} = \frac{E}{L_{(1)}} \begin{bmatrix} -l_1 & -m_1 & l_1 & m_1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix}$$

$$\sigma^{(1)}_{[1-2]} = \frac{210\times10^9}{5} \begin{bmatrix} 0.707 & -0.707 & -0.707 & 0.707 \end{bmatrix} \begin{Bmatrix} -3.448\times10^{-3} \\ -6.896\times10^{-3} \\ 0 \\ 0 \end{Bmatrix}$$

$$= 102.4\,\text{MPa}\ [T]$$

$$\sigma^{(2)}_{[1-3]} = \frac{E}{L_2} \begin{bmatrix} -l_2 & -m_2 & l_2 & m_2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_5 \\ u_6 \end{bmatrix}$$

$$= \frac{210\times10^9}{10} \begin{bmatrix} 1 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} -3.448\times10^{-3} \\ -6.896\times10^{-3} \\ 0 \\ 0 \end{bmatrix}$$

$$= -72\,\text{MPa}\ [C]$$

$\sigma = P/A$

$f_{1-2} = 102.4 \times 5\times10^{-4}$ ($\text{MPa} \times 10^6$)

$= 51.2\,\text{kN}$

$f_{1-3} = -72.[illegible] \times [illegible]$ ($\text{MPa} \times 10^6$)

$= 55.[illegible]\,\text{kN}$

Note : can show equilibrium at node 1

$$F_s = \left[2000\,\text{kN/m}\right] \times \left[6.896\times10^{-3}\,\text{m}\right] = 13.792\,\text{kN}$$

$f_{1-3} = 35.6\,\text{kN}$

$f_{1-2} = 51.2\,\text{kN}$

$\Sigma F_y = 0$

$-50 + 13.79 + 36.198 = 0$

For a tapered plate of uniform thickness t=10mm as shown in Figure.1, find the displacements at the nodes by meshing into two element model. The bar has mass density $\rho = 7800 kg/m^3$, $E = 2\times10^5 MN/m^2$. In addition to self-weight, the plate is subjected to a point load P=10kN at its center. Also determine the reaction force at the support.

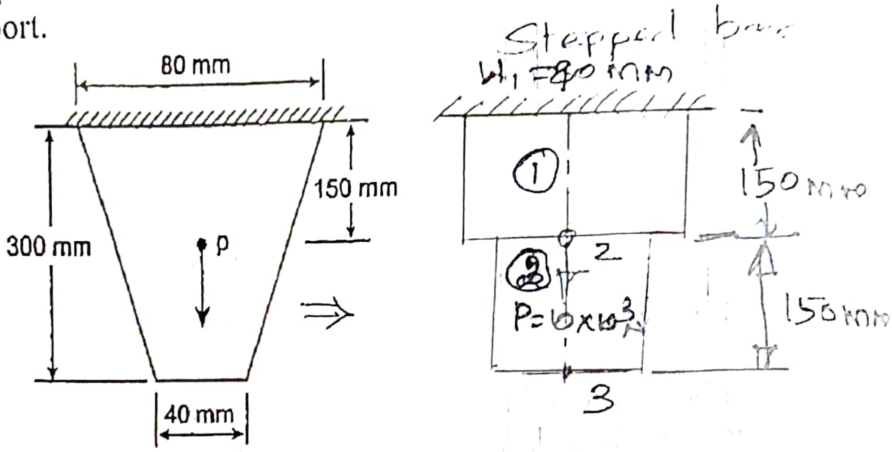


Figure .1

Solution

Area at node 1, A_1

$= \text{Width} \times \text{thickness} = W_1 \times t_1$

$= 80 \times 10$

$A_1 = 800 \text{ mm}^2$

Areat at node 2, A_2

$A_2 = \left[\dfrac{W_1 + W_3}{2}\right] \times t_2$

$= \left[\dfrac{80+40}{2}\right] \times 10$

$A_2 = 600 \text{mm}^2$

$[t_1, t_2, t_3 = 10 \text{ mm}]$

Area at node 3, A_3

$A_3 = W_3 \times t_3 = 40 \times 10$

$= 400 \text{mm}^2$

Average area of element (1) $A_1^{(1)} =$

$= \dfrac{\text{Area at node 1} + \text{Area at nod 2}}{2}$

$= \dfrac{800+600}{2}$

$A_1^{(1)} = 700 \text{ mm}^2$

Average Area of element (2) $A_2^{(2)}$

$= \dfrac{A_2 + A_3}{2}$

$= \dfrac{600+400}{2}$

$A_2^{(2)} = 500 \text{ mm}^2$

Mass density $\rho = 7800\ kg/m^3 = 7800 \times 9.81\ N/m^3$
$= 76518\ N/m^3 = 76518 \times 10^{-9}\ N/mm^3$
$= 7.6518 \times 10^{-5}\ N/mm^3$

Young's Moduls $E = 2 \times 10^5\ MN/m^2$
$= 2 \times 10^5 \times 10^6\ N/m^2$
$= 2 \times 10^5 \times 10^6 \times 10^{-6}\ N/mm^2$
$= 2 \times 10^5\ N/mm^2$

Stiffness matrix for element ①

$$K_1 = \frac{A_1^{(1)} E}{L_1}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$$

$$= \frac{700 \times 2 \times 10^5}{150}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$$

$$= 4.666 \times 2 \times 10^5\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$$

$$= 10^5 \begin{array}{cc} \begin{array}{cc} 1 & 2 \end{array} & \\ \begin{bmatrix}9.332 & -9.332\\ -9.332 & 9.332\end{bmatrix} & \begin{array}{c}1\\2\end{array}\end{array}$$

Stiffness matrix for element ②

$$K_2 = \frac{A_2^{(2)} E}{L_2}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$$

$$= \frac{500 \times 2 \times 10^5}{150}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$$

$$= 6.666 \times 10^5\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$$

$$= 10^5 \begin{array}{c} \begin{array}{cc} 2 & 3 \end{array} \\ \begin{bmatrix}6.666 & -6.666\\ -6.666 & 6.666\end{bmatrix}\end{array}$$

Global matrix $[K] = K_1 + K_2$

$$= 10^5 \times \begin{bmatrix}9.332 & -9.332 & 0\\ -9.332 & \boxed{9.332+6.666} \to 15.998 & -6.666\\ 0 & -6.666 & 6.666\end{bmatrix}$$

Displacement Vector $U = \begin{bmatrix}u_1\\ u_2\\ u_3\end{bmatrix}$

2/5

Body force Vector $\{F\}_f = \frac{\rho A L}{2}\begin{bmatrix}1\\1\end{bmatrix}$

Force Vector For element (1)

$$\begin{Bmatrix}F_1\\F_2\end{Bmatrix} = \frac{\rho_1 A_1^{(1)} L_1}{2}\begin{bmatrix}1\\1\end{bmatrix}$$

$$= 7.6518\times10^{-5}\times700\times150\begin{bmatrix}1\\1\end{bmatrix}$$

$$= \begin{array}{c}\begin{bmatrix}4.017\\4.017\end{bmatrix}\end{array}\begin{array}{c}1\\2\end{array}$$

Force Vector for element (2)

$$\begin{bmatrix}F_2\\F_3\end{bmatrix} = \frac{\rho_2 A_2^{(2)} L_2}{2}\begin{bmatrix}1\\1\end{bmatrix}$$

$$= 7.6518\times10^{-5}\times500\times150\begin{bmatrix}1\\1\end{bmatrix}$$

$$= \begin{array}{c}\begin{bmatrix}2.869\\2.869\end{bmatrix}\end{array}\begin{array}{c}2\\3\end{array}$$

Global force vector

$$\begin{Bmatrix}F_1\\F_2\\F_3\end{Bmatrix} = \begin{Bmatrix}4.017\\4.017+2.869\\2.869\end{Bmatrix} = \begin{Bmatrix}4.017\\6.886\\2.869\end{Bmatrix}$$

Assemble the finite element equation $[K]\{u\}=\{F\}$

$$10^5\begin{bmatrix}9.332 & -9.332 & 0\\-9.332 & 15.998 & -6.666\\0 & -6.666 & 6.666\end{bmatrix}\begin{bmatrix}u_1\\u_2\\u_3\end{bmatrix} = \begin{bmatrix}4.017\\6.886\\2.869\end{bmatrix}$$

Apply the boundary condition i.e at node 1 displace $u_1 = 0$.

A point load of 10×10^3 N is acting at node 2, so add 10,000 N in F_2 vector

$$10^5 \begin{bmatrix} 9.332 & -9.332 & 0 \\ -9.332 & 15.998 & -6.666 \\ 0 & -6.666 & 6.666 \end{bmatrix} \begin{bmatrix} 0 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 4.017 \\ 10006.886 \\ 2.869 \end{bmatrix}$$

In the above equation $u_1 = 0$, so neglect first row and first column of [K] matrix. The reduced equation is.

$$10^5 \begin{bmatrix} 15.998 & -6.666 \\ -6.666 & 6.666 \end{bmatrix} \begin{bmatrix} u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 10006.886 \\ 2.869 \end{bmatrix}$$

$$[15.998 U_2 - 6.666 U_3] 10^5 = 100006.886 \quad \text{--- (1)}$$

$$[-6.666 U_2 + 6.666 U_3] 10^5 = 2.869 \quad \text{--- (2)}$$

Solve above equation $u_2 = 0.01073$ mm

$u_3 = 0.01073$ mm

Reaction force $\{R\} = [K]\{U\} - \{F\}$

$$= 10^5 \begin{bmatrix} 9.332 & -9.332 & 0 \\ -9.332 & 15.998 & -6.666 \\ 0 & -6.666 & 6.666 \end{bmatrix} \begin{bmatrix} 0 \\ 0.01073 \\ 0.01073 \end{bmatrix} - \begin{bmatrix} 4.017 \\ 10,006.886 \\ 2.869 \end{bmatrix}$$

$$= \begin{bmatrix} -10,000 \\ 10,000 \\ 0 \end{bmatrix} - \begin{bmatrix} 4.017 \\ 10,006.886 \\ 2.869 \end{bmatrix}$$

$$\begin{Bmatrix} R_1 \\ R_2 \\ R_3 \end{Bmatrix} = \begin{Bmatrix} -10004.017 \\ -6.886 \\ -2.869 \end{Bmatrix} N$$

Reaction force is equivalent and opposite to applied force.

Verification $R_1 + R_2 + R_3$

$= -10004.017 - 6.886 - 2.869$

$= -10013.772$ N

Applied force $= 4.017 + 10,006.886 + 2.869$

$= 10013.772$ N

Result: Displacement $U_1 = 0$

$U_2, U_3 = 0.01073$ mm.

Reaction force at the suppor

$R_1 = -10004.017$ N

A compound axial member is subjected to the loads shown in Fig. 1.1. Given, $E_1 = 50MN/m^2$, $E_2 = 100MN/m^2$, $L_1 = 0.5m$, $L_2 = 1m$, $A_1 = 20cm^2$, $A_2 = 10cm^2$, find (i) reaction force at points 1 (F1) and (ii) displacements at nodes 2 and 3 (u_2 and u_3) using two bar elements model.

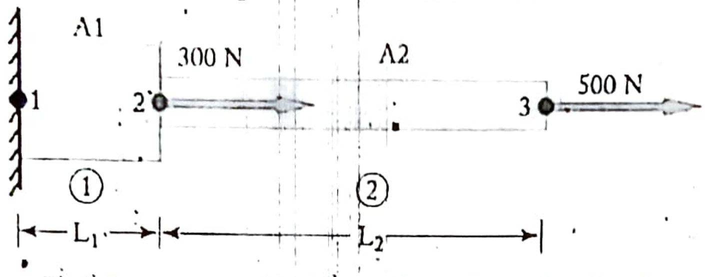
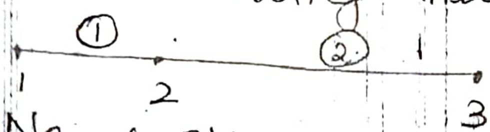


Fig1.1 A compound axial member subjected to loads.

Solution.

Pre-processing FEA model



No of Elements = 2
number of nodes = 3

The stiffness matrix of each element is computed from $[K]^e = \frac{AE}{L}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$

For element (1), the stiffness matrix is

$$[K]^{(1)} = 0.002 \times 50 \times 10^6 \begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$$

$$= 2 \times 10^5 \begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$$

$$= 10^5 \begin{bmatrix}2 & -2\\ -2 & 2\end{bmatrix}$$

For element (2), the stiffness matrix is

$$[K]^{(2)} = 0.001 \times 100 \times 10^6 \begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$$

$$= 1 \times 10^5 \begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$$

$$= 10^5 \begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$$

The final global matrix of stiffness matrix

$$[K]^{(G)} = 10^5 \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix}2 & -2 & 0\\ -2 & 2+1 & -1\\ 0 & -1 & 1\end{bmatrix} \end{array} \begin{array}{c} 1 \\ 2 \\ 3 \end{array}$$

1/2

Displacement $\{u\} = \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix}$

load/Force matrix $\{F\} = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 300 \\ 500 \end{Bmatrix}$

Finite Element Equation $[K]\{U\} = \{F\}$

$$10^5 \begin{bmatrix} 2 & -2 & 0 \\ -2 & 3 & -1 \\ 0 & -1 & 0 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 300 \\ 500 \end{Bmatrix}$$

Apply the fixed boundary condition at node 1 and applying the external forces at nodes 2 and 3,

$$10^5 \begin{bmatrix} \cancel{2} & \cancel{-2} & \cancel{0} \\ \cancel{-2} & 3 & -1 \\ \cancel{0} & -1 & 0 \end{bmatrix} \begin{Bmatrix} \cancel{0} \\ u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} \cancel{0} \\ 300 \\ 500 \end{Bmatrix}$$

Using the matrix partitioning to solve for u2 and u_3, we have

$$10^5 \begin{bmatrix} 3 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 300 \\ 500 \end{bmatrix}$$

$$\begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} 0.004 \\ 0.009 \end{Bmatrix} m$$

Post-processing: using the First row of the global matrix to find (R_1), we have

$$R_1 = -2 \times 10^5 \times 0.004 = -800N$$

check for Equilibrium

Action Forces = - Reaction forces. $R_1 = -800N$

$F_2 + F_3 = 300 + 500 = 800N$