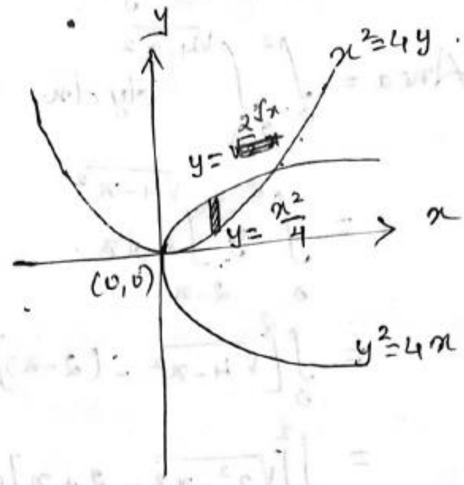
AREA AS A DOUBLE INTEGRAL

1) Find the area include between the curve $y^2 = 4x$ and $x^2 = 4y$



Gn: $y^2 = 4x$ & $x^2 = 4y$

$\Rightarrow x = \frac{y^2}{4}$

$x^2 = 4y$

$\Rightarrow \frac{y^4}{16} = 4y$

$\Rightarrow y^3 = 4^3$

$\Rightarrow y = 4$

$y = 4 \Rightarrow y^2 = 4x$

$16 = 4x$

$\boxed{4 = x}$

$\therefore$ The pt. is (4,4)

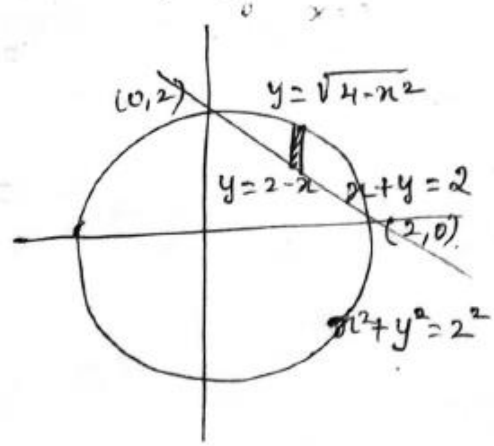
$$\int_0^4 \int_{\frac{x^2}{4}}^{2\sqrt{x}} dy\, dx$$

$$= \int_0^4 y \Big]_{\frac{x^2}{4}}^{2\sqrt{x}} dx = \int_0^4 \left(2x^{1/2} - \frac{x^2}{4}\right) dx$$

$$= \frac{2x^{3/2}}{3/2} - \frac{x^3}{12} \Big]_0^4 = \frac{4}{3}(4)^{3/2} - \frac{4^3}{12}$$

$$= \frac{32 - 16}{3} = \frac{16}{3} \text{ Sq. units.}$$

(2) Find the small area bounded by $y = 2 - x$ & $x^2 + y^2 = 4$



$y = 2 - x$ & $x^2 + y^2 = 4$

$x^2 + (2-x)^2 = 4$

$x^2 + \cancel{4} + x^2 - 4x = \cancel{4}$

$2x^2 - 4x = 0$

$\Rightarrow x^2 - 2x = 0$

$\Rightarrow x(x-2) = 0$

$x = 0$ or $x = 2$.

when $x = 0 \Rightarrow y = 2$.

$x = 2 \Rightarrow y = 0$

$$\text{Area} = \int_0^2 \int_{2-x}^{\sqrt{4-x^2}} dy\,dx$$

$$= \int_0^2 \left[y \right]_{2-x}^{\sqrt{4-x^2}} dx$$

$$= \int_0^2 \left[\sqrt{4-x^2} - (2-x)\right] dx$$

$$= \int_0^2 \left[\sqrt{2^2-x^2} - 2 + x\right] dx$$

$$= \left[\left[\frac{x}{2}\sqrt{2^2-x^2} + \frac{4}{2}\sin^{-1}\left(\frac{x}{2}\right)\right] - 2x + \frac{x^2}{2}\right]_0^2$$

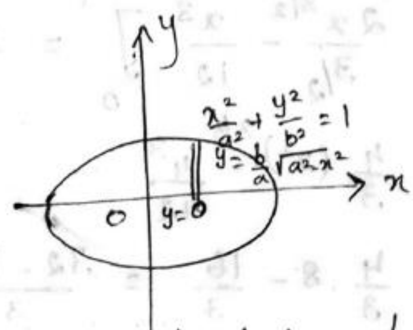
$$= \left[0 + 2\sin^{-1}(1) - 4 + 2\right]$$

$$= 2 \cdot \frac{\pi}{2} - 2$$

$$= \pi - 2 \text{ sq. units}$$

(3) Find the area enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

In the I quadrant of the ellipse, y varies from 0 to $\frac{b}{a}\sqrt{a^2 - x^2}$ and x varies from 0 to a



Area of the ellipse $= 4$ (area of the first quadrant)

$$= 4 \int_0^a \int_0^{\frac{b}{a}\sqrt{a^2-x^2}} dy\, dx$$

$$= 4 \int_0^a \left[y \right]_0^{\frac{b}{a}\sqrt{a^2-x^2}} dx$$

$$= 4 \int_0^a \frac{b}{a}\sqrt{a^2-x^2}\, dx$$

$$= \frac{4b}{a} \int_0^a \sqrt{a^2-x^2}\, dx$$

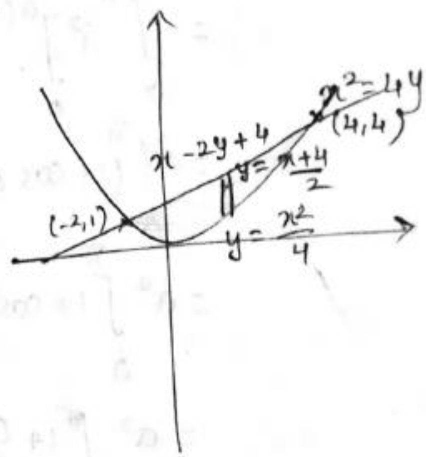
$$= \frac{4b}{a} \left[\frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} \right]_0^a$$

$$= \frac{4b}{a}\left[\frac{a}{2}(0) + \frac{a^2}{2}\sin^{-1}(1)\right]$$

$$= 4\frac{b}{a}\left[\frac{a^2}{2}\cdot\frac{\pi}{2}\right]$$

$$= \pi ab \text{ sq.units}$$

(4) Find the area between the parabola $x^2 = 4y$ & the st. line $x - 2y + 4 = 0$



Given:

$x^2 = 4y$ & $x - 2y + 4 = 0$

$y = \frac{x^2}{4}$

$\Rightarrow x - \cancel{2}\frac{x^2}{\cancel{4}2} + 4 = 0$

$\Rightarrow 2x - x^2 + 8 = 0$

$\Rightarrow x^2 - 2x - 8 = 0$

$\Rightarrow (x-4)(x+2) = 0$

$\Rightarrow x = 4$ or $x = -2$.

when $x = 4 \Rightarrow y = 4$

$x = -2 \Rightarrow y = 1$

$\therefore$ The pts are $(4,4)$ & $(-2,1)$

$\therefore$ The pts are $(4,4)$ & $(-2,1)$

$$\int_{-2}^{4}\int_{\frac{x^2}{4}}^{\frac{x+4}{2}} dy\,dx.$$

$$= \int_{-2}^{4} y\Big]_{x^2/4}^{\frac{x+4}{2}} dx = \int_{-2}^{4} \frac{x+4}{2} - \frac{x^2}{4}\,dx.$$

$$= \left[\frac{x^2}{4} + \frac{4x}{2} - \frac{x^3}{12}\right]_{-2}^{4} = \left[\frac{16}{4} + \frac{16}{2} - \frac{64}{12} - \left[\frac{4}{4} - \frac{8}{2} + \frac{8}{12}\right]\right]$$

$$= \left[4 + 8 - \frac{16}{3} - \left[1 - 4 + \frac{2}{3}\right]\right] = 9 \text{ sq. units}.$$