SNS COLLEGE OF TENHNOLOGY

(AN AUTONOMOUS INSTITUTION)

UNIT-V

LAPLACE TRANSFORMS

PART B QUESTIONS & ANSWERS

1 .Verify Initial Value theorem for $f(t)=e^{-t}\sin t$.

Solution:

Initial Value theorem: $\lim_{t\to 0} f(t) = \lim_{s\to\infty} sF(s)$

Now,

$$\lim_{t\to 0} f(t) = \lim_{t\to 0}\left[e^{-t}\sin t\right] = e^{0}\sin 0 = 1*0 = 0$$

$$\lim_{s\to\infty} sF(s) = \lim_{s\to\infty} s.L\left[e^{-t}\sin t\right] = \lim_{s\to\infty} s\left[\frac{1}{s^2+1}\right]_{s\to s+1}$$

$$= \lim_{s\to\infty} s\left[\frac{1}{(s+1)^2+1}\right] = \lim_{s\to\infty} s\left[\frac{1}{s^2+2s+2}\right]$$

$$= \lim_{s\to\infty} \frac{s}{s^2\left[1+\frac{2}{s}+\frac{2}{s^2}\right]} = \lim_{s\to\infty} \frac{1}{s\left[1+\frac{2}{s}+\frac{2}{s^2}\right]} = \frac{1}{\infty} = 0$$

$\therefore$ **LHS = RHS.**

Hence verified.

2. Find the inverse laplace transform of $\log\left[\frac{s+1}{s}\right]$.

Solution:

$$L^{-1}\left[\log\left(\frac{s+1}{s}\right)\right] = -\frac{1}{t}\left[L^{-1}\frac{d}{ds}\log\left(\frac{s+1}{s}\right)\right] = -\frac{1}{t}L^{-1}\left[\frac{d}{ds}\left(\log(s+1)-\log s\right)\right]$$

$$= -\frac{1}{t}L^{-1}\left[\frac{1}{s+1}-\frac{1}{s}\right] = -\frac{1}{t}\left[e^{-t}-1\right] = \frac{1-e^{-t}}{t}$$

$$L^{-1}\left[\log\left(\frac{s+1}{s}\right)\right] = \frac{1-e^{-t}}{t}$$

3.Find $L[3e^{5t}+5\cos t]$.

Solution:

$$L[3e^{5t}+5\cos t] = 3L[e^{5t}]+5L[\cos t]$$

$$= 3\frac{1}{s-5}+5\frac{s}{s^2+1}$$

4. If L[F(t)]=F(s),prove that $L[F(at)] = \frac{1}{a}F\left(\frac{s}{a}\right)$

Proof:

By definition, we have

$$L[f(at)] = \int_0^\infty e^{-st} f(at)dt$$

put $at = x \Rightarrow t = \frac{x}{a}; dt = \frac{dx}{a}$

$$L[f(at)] = \int_0^\infty e^{-s(x/a)} f(x)\frac{dx}{a}$$

$$= \frac{1}{a}\int_0^\infty e^{-\left(\frac{s}{a}\right)x} f(x)dx$$

$$= \frac{1}{a}\int_0^\infty e^{-\left(\frac{s}{a}\right)t} f(t)dt$$

$$L[f(at)] = \frac{1}{a}F\left(\frac{s}{a}\right)$$

5. Find $L[e^{-at}\sin bt]$

$$L[e^{-at}\sin bt] = L[\sin bt]_{s\to s+a}$$

$$= \left[\frac{b}{s^2+b^2}\right]_{s\to s+a}$$

$$= \frac{b}{(s+a)^2+b^2}$$

$$= \frac{b}{s^2+a^2+2sa+b^2}$$

6. Find $L^{-1}\left[\frac{1}{(s+2)^3}\right]$

$$L^{-1}\left[\frac{1}{(s+2)^3}\right] = e^{-2t}L^{-1}\left[\frac{1}{s^3}\right]$$

$$= e^{-2t}\left[\frac{t^2}{2!}\right]$$

$$L^{-1}\left[\frac{1}{(s+2)^3}\right] = e^{-2t}\frac{t^2}{2!}$$

7.. If $L[f(t)] = F(s)$ and $g(t) = \begin{cases} f(t-a), t > a \\ 0, t < a \end{cases}$ then prove that $L[g(t)] = e^{-as}F(s)$

$$L[g(t)] = \int_0^\infty e^{-st}g(t)dt = \int_0^a e^{-st}g(t)dt + \int_a^\infty e^{-st}g(t)dt$$

$$= \int_0^a e^{-st}0dt + \int_a^\infty e^{-st}f(t-a)dt$$

Put $t - a = x; t = a, x = 0$
$dt = dx; t = \infty, x = \infty$

$$L[g(t)] = \int_0^\infty e^{-s(x+a)}f(x)dx = \int_0^\infty e^{-sx}e^{-sa}f(x)dx$$

$$= e^{-sa}\int_0^\infty e^{-sx} f(x)dx$$

$$= e^{-sa}\int_0^\infty e^{-st} f(t)dt$$

$$= e^{-sa}F(s)$$

8. Find $L[te^{-2t}\cos 2t]$

$$L[te^{-2t}\cos 2t] = -\frac{d}{ds}\left\{L[te^{-2t}\cos 2t]\right\}$$

$$= -\frac{d}{ds}\{L[\cos 2t]\}_{s\to s+a}$$

$$= -\frac{d}{ds}\left[\frac{s}{s^2+2^2}\right]_{s\to s+a}$$

$$= -\left[\frac{(s^2+4)(1)-s(2s)}{(s^2+4)^2}\right]_{s\to s+a}$$

$$= -\left[\frac{s^2+4-2s^2}{(s^2+4)^2}\right]_{s\to s+a}$$

$$= -\left[\frac{4-s^2}{(s^2+4)^2}\right]_{s\to s+a}$$

$$= -\left[\frac{-(s^2-4)}{(s^2+4)^2}\right]_{s\to s+a} = \left[\frac{(s^2-4)}{(s^2+4)^2}\right]_{s\to s+a}$$

$$= \left[\frac{(s+2)^2-4)}{[(s+2)^2+4]^2}\right] = \frac{s^2+2s}{[s^2+2s+s]^2}$$

Hence $L[te^{-2t}\cos 2t] = \frac{s^2+2s}{[s^2+2s+s]^2}$

9. Prove that $\int_0^\infty te^{-3t}\sin t dt = \frac{3}{50}$

$$\int_0^\infty te^{-3t}\sin t dt = \left[\int_0^\infty e^{-st}(t\sin t)dt\right]_{s=3}$$

$$= \{L[t\sin t]\}_{s=3} = -\frac{d}{ds}L[\sin t]_{s=3}$$

$$= -\frac{d}{ds}\left[\frac{1}{s^2+1}\right]_{s=3} = -\left[\frac{(s^2+1)(0)-1(2s)}{(s^2+1)^2}\right]_{s=3}$$

$$= -\left[\frac{-2s}{(s^2+1)^2}\right]_{s=3} = \left[\frac{2s}{(s^2+1)^2}\right]_{s=3} = \frac{6}{(10)^2} = \frac{3}{50}$$

Hence $\int_0^\infty te^{-3t}\sin t dt = \frac{3}{50}$

10. Using Laplace Transforms of derivatives find $L[e^{-at}]$

By Laplace Transforms of derivatives,

$$L\{f'(t)\} = sL\{f(t)\} - f(0)$$

$$L\{f''(t)\} = s^2L\{f(t)\} - sf(0) - f'(0)$$

$$L[e^{-at}]; \quad \begin{matrix} f(t)e^{-at} =, f(0)e^{0} = 1 \\ f'(t)e^{-at}(-a) \end{matrix}$$

$$\therefore L[f'(t)] = sL[f(t)] - f(0)$$

$$\therefore L[-ae^{-at}] = sL[e^{-at}] - 1$$

$$= s\left(\frac{1}{s+a}\right) - 1 = \frac{s}{s+a} - 1 = \frac{-a}{s+a}$$

$$L[-ae^{-at}] = \frac{-a}{s+a}$$

$$-aL[e^{-at}] = \frac{-a}{s+a}$$

$$L[e^{-at}] = \frac{1}{s+a}$$

11. Verify the initial value theorem for 3+4 cos2t

Theorem:

$$\lim_{t\to 0} f(t) = \lim_{s\to\infty} sF(s)$$

$$\lim_{t\to 0}(3+4\cos 2t) = 3+4\left[\lim_{t\to 0}\cos 2t\right] = 3+4[\cos 0] = 7$$

$$\lim_{s\to\infty} sF(s) = \lim_{s\to\infty} sL[3+4\cos 2t] = \lim_{s\to\infty} s[L(3)+4L(\cos 2t)]$$

$$= \lim_{s\to\infty} s[3L(1)+4L(\cos 2t)] = \lim_{s\to\infty} s\left[3.\frac{1}{s} + 4\frac{s}{s^2+4}\right]$$

$$= \lim_{s\to\infty}\left[3+4\frac{s^2}{s^2+4}\right] = \lim_{s\to\infty}\left\{3+4\left[\frac{1}{1+\frac{4}{s^2}}\right]\right\}$$

$$= 3+4\lim_{s\to\infty}\left[\frac{1}{1+\frac{4}{s^2}}\right] = 3+4\left[\frac{1}{1+\frac{4}{\infty}}\right]$$

$$= 3+4\left[\frac{1}{1+0}\right] = 7$$

L.H.S = R.H.S

12. Find $L\left[\frac{1}{\sqrt{\pi t}}\right]$

$$L\left[\frac{1}{\sqrt{\pi t}}\right] = \frac{1}{\sqrt{\pi}} L\left[t^{-\frac{1}{2}}\right] = \frac{1}{\sqrt{\pi}}\left[\frac{\left|\overline{-\frac{1}{2}+1}\right.}{s^{-\frac{1}{2}+1}}\right]$$

$$= \frac{1}{\sqrt{\pi}}\left[\frac{\left|\overline{\frac{1}{2}}\right.}{s^{1/2}}\right] = \frac{1}{\sqrt{\pi}}\left[\frac{\sqrt{\pi}}{s^{1/2}}\right] = \frac{1}{s^{1/2}}$$

$$\therefore L\left[\frac{1}{\sqrt{\pi t}}\right] = \frac{1}{s^{1/2}}$$

13. State Initial value theorem on Laplace Transforms.

If the Laplace transforms of f(t) and f '(t) exist and $L[f(t)] = F(s)$ then $\lim_{t\to 0}[f(t)] = \lim_{s\to\infty}[sF(s)]$

14. Find . $L^{-1}\left[\frac{4s+13}{s^2+5}\right]$

$$L^{-1}\left[\frac{4s+13}{s^2+5}\right] = L^{-1}\left[\frac{4s}{s^2+5}\right] + L^{-1}\left[\frac{13}{s^2+5}\right]$$

$$= 4L^{-1}\left[\frac{s}{s^2+(\sqrt{5})^2}\right] + 13L^{-1}\left[\frac{1}{s^2+(\sqrt{5})^2}\right]$$

$$= 4\cos\sqrt{5}t + \frac{13}{\sqrt{5}}L^{-1}\left[\frac{\sqrt{5}}{s^2+(\sqrt{5})^2}\right]$$

$$= 4\cos\sqrt{5}t + \frac{13}{\sqrt{5}}\sin\sqrt{5}t$$

15. State convolution theorem on Laplace Transforms.

If f(t) and g(t) are two functions defined for $t \geq 0$, then

$$L[(f*g)(t)] = L[f(t)].L[g(t)]$$

ie., $L[(f*g)(t)] = F(s).G(s)$ where $L[f(t)] = F(s)$ and $L[g(t)] = G(s)$

16. Find $L\left[\frac{\cos at - \cos bt}{t}\right]$

$$L\left[\frac{\cos at - \cos bt}{t}\right] = \int_s^\infty L[\cos at - \cos bt]ds$$

$$= \int_s^\infty \{L[\cos at]L[\cos bt]\}ds = \int_s^\infty \left\{\left(\frac{s}{s^2+a^2}\right) - \left(\frac{s}{s^2+b^2}\right)\right\}ds$$

$$= \left[\frac{1}{2}\log(s^2+a^2) - \frac{1}{2}\log(s^2+b^2)\right]_s^\infty = \frac{1}{2}\left[\log(s^2+a^2) - \log(s^2+b^2)\right]_s^\infty$$

$$= \frac{1}{2}\left[\log\left(\frac{s^2+a^2}{s^2+b^2}\right)\right]_s^\infty = \frac{1}{2}\left[\log\left(\frac{s^2(1+a^2/s^2)}{s^2(1+b^2/s^2)}\right)\right]_s^\infty$$

$$= \frac{1}{2}\left[\log\left(\frac{1+a^2/s^2}{1+b^2/s^2}\right)\right]_s^\infty = \frac{1}{2}\left\{\log 1 - \log\left(\frac{s^2+a^2}{s^2+b^2}\right)\right\}$$

$$= -\frac{1}{2}\left\{0 - \log\left(\frac{s^2+a^2}{s^2+b^2}\right)\right\} = \frac{1}{2}\left\{\log\left(\frac{s^2+b^2}{s^2+a^2}\right)\right\}$$

17. Find the inverse Laplace transform of $\left[\frac{s+2}{(s+3)(s^2+4)}\right]$

Now $\frac{s+2}{(s+3)(s^2+4)} = \frac{A}{s+3} + \frac{Bs+c}{s^2+4}$

$$s+2 = A(s^2+4)+(Bs+c)(s+3)$$

Put s = -3

$$-1 = A(9+4)+\left[B(-3)+c\right](0) \Rightarrow A = \frac{-1}{13}$$

Put s = 0

$$2 = 4A + c(3) \Rightarrow c = \frac{10}{13}$$

Put s = 1

$$3 = A(5)+\left[B+c\right](4) \Rightarrow B = \frac{1}{13}$$

$$\therefore \frac{s+2}{(s+3)(s^2+4)} = -\frac{1}{13}\frac{1}{(s+3)}+\frac{s}{13(s^2+4)}+\frac{10}{13}\frac{1}{(s^2+4)}$$

$$L^{-1}\left[\frac{s+2}{(s+3)(s^2+4)}\right] = L^{-1}\left[-\frac{1}{13}\frac{1}{(s+3)}+\frac{s}{13(s^2+4)}+\frac{10}{13}\frac{1}{(s^2+4)}\right]$$

$$= -\frac{1}{13}L^{-1}\left[\frac{1}{(s+3)}\right]+\frac{1}{13}L^{-1}\left[\frac{s}{(s^2+4)}\right]+\frac{10}{13}L^{-1}\left[\frac{1}{(s^2+4)}\right]$$

$$= -\frac{1}{13}e^{-st}+\frac{1}{13}\cos 2t+\frac{10}{13}\frac{\sin 2t}{2}$$

$$= \frac{1}{13}\left[\cos 2t + 5\sin 2t - e^{-st}\right]$$

18. Find the inverse Laplace transform of $\log\left[\frac{s-5}{s^2+9}\right]$

$$L^{-1}\left[\log\left[\frac{s-5}{s^2+9}\right]\right] = L^{-1}[F(s)] = -\frac{1}{t}L^{-1}[F'(s)]$$

Now,

$$F(s) = \log\left[\frac{s-5}{s^2+9}\right] = \log(s-5)-\log(s^2+9)$$

$$F'(s) = \frac{1}{s-5}-\frac{1}{s^2+9}.2s$$

$$L^{-1}\left[F'(s)\right] = L^{-1}\left[\frac{1}{s-5}-\frac{2s}{s^2+9}\right] = L^{-1}\left[\frac{1}{s-5}\right]-2L^{-1}\left[\frac{s}{s^2+9}\right]$$

$$= e^{5t}-2\cos 3t$$

Now,

$$L^{-1}\left[F'(s)\right] = -\frac{1}{t}L^{-1}\left[F'(s)\right] = -\frac{1}{t}\left[e^{5t}-2\cos 3t\right] = \frac{2\cos 3t - e^{5t}}{t}$$

19. If $L[f(t)] = \frac{s+2}{s^2+4}$, find the value of $\int_0^\infty f(t)dt$

$$\left\{\int_0^\infty e^{-st}f(t)dt\right\}_{s=0} = \left\{L[f(t)\right\}_{s=0} = \left\{\frac{s+2}{s^2+4}\right\}_{s=0} = \frac{2}{4} = \frac{1}{2}$$

20. State the first Shifting theorem on Laplace transforms

If $L[f(t)] = F(s)$ then

$$(i)L[e^{at}f(t)] = F(s-a)$$
$$(ii)L[e^{-at}f(t)] = F(s+a)$$

21. State and prove second Shifting theorem on Laplace transforms.

If $L[f(t)] = F(s)$ then $L[f(t-a)u(t-a)] = e^{-as}F(s)$.

Proof:

$$L[f(t-a)u(t-a)] = \int_0^\infty e^{-st}f(t-a)u(t-a)dt$$
$$= \int_0^a e^{-st}(0)dt + \int_a^\infty e^{-st}f(t-a)dt$$

[By defn. of unit step function]

$$= \int_0^\infty e^{-s(u+a)}f(u)du$$

put, $t-a=u$
$dt = du$
$t = a \Rightarrow u = 0$
$t = \infty \Rightarrow u = \infty$

$$= e^{-as}\int_0^\infty e^{-su}f(u)du$$
$$= e^{-as}F(s)$$

22. Define unit impulse function

The unit impulse function is defined by

$$\delta(t-a) = \begin{cases} \infty, t = a \\ 0, t \neq a \end{cases}$$

such that $\int_{-\infty}^{\infty} \delta(t-a)dt = 1$. It exists only at t = a at which it is infinitely great and is denoted by $\delta(t-a)$.

23. State the Laplace transforms of periodic function with period transforms.

The Laplace transforms of a periodic function f(t) with period 'p' given by,

$$L[f(t)] = \frac{1}{1-e^{-Ts}}\int_0^T e^{-st}f(t)dt$$

24. Find $L\left[\sinh^2 2t\right]$

We know that

$$\sinh^2 2t = \frac{1}{2}[\cosh 4t - 1]$$
$$\left[\cosh 2(2t) = 1 + 2\sinh^2 2t\right]$$

Now,

$$L\left[\sinh^2 2t\right] = \frac{1}{2}L[\cosh 4t - 1]$$
$$= \frac{1}{2}\{L[\cosh 4t] - L[1]\}$$
$$= \frac{1}{2}\left\{\frac{s}{s^2-16} - \frac{1}{s}\right\} = \frac{8}{s(s^2-16)}$$

25. Find $L^{-1}\left[\frac{1+e^{-s}}{s}\right]$

[By Second Shifting property $L[e^{-as}F(s)] = f(t-a)u(t-a)$]

We can write

$$L^{-1}\left[\frac{1+e^{-s}}{s}\right] = L^{-1}\left[\frac{1}{s}\right] + L^{-1}\left[\frac{e^{-s}}{s}\right] = 1 + L^{-1}\left[e^{-s}\frac{1}{s}\right]$$

Consider $L^{-1}\left[e^{-s}\frac{1}{s}\right]$

Here $F(s) = \frac{1}{s}$

$$L^{-1}[F(s)] = L^{-1}\left[\frac{1}{s}\right] = 1 = f(t)$$

$$\therefore L^{-1}\left[e^{-s}\frac{1}{s}\right] = 1.u(t-1)$$

$$\therefore L^{-1}\left[\frac{1+e^{-s}}{s}\right] = 1 + u(t-1)$$

26. Evaluate $\int_0^{\infty} \frac{e^{-t}\sin t}{t} dt$ using laplace transforms.

$$\int_0^{\infty} \frac{e^{-t}\sin t}{t} dt = \left\{\int_0^{\infty} e^{-st} f(t) dt\right\}_{s=1} = \{L[f(t)]\}_{s=1}$$

Now, $L[f(t)] = L\left[\frac{\sin t}{t}\right] = \int_s^{\infty} L[\sin t] ds$

$$= \int_s^{\infty} \frac{1}{s^{2+1}} ds = \left[\tan^{-1}(s)\right]_s^{\infty}$$

$$= \tan^{-1}(\infty) - \tan^{-1}(s)$$

$$= \frac{\pi}{2} - \tan^{-1}(s) = \cot^{-1} s$$

$$\{L[f(t)]\}_{s=1} = \left\{\cot^{-1} s\right\}_{s=1} = \frac{\pi}{4}$$

$$\int_0^{\infty} \frac{e^{-t}\sin t}{t} dt = \frac{\pi}{4}$$

27. Find the Laplace transforms of impulse function.

$$L[\delta(t-a)] = \lim_{\xi \to 0}\left[\delta_{\xi}(t-a)\right] = \lim_{\xi \to 0}\int_0^{\infty} e^{-st}\delta_{\xi}(t-a) dt$$

$$= \lim_{\xi\to 0}\int_0^{\infty} e^{-st}\delta_\xi(t-a)dt + \lim_{\xi\to 0}\int_a^{a+\xi} e^{-st}\delta_\xi(t-a)dt + \lim_{\xi\to 0}\int_{a+\xi}^{\infty} e^{-st}\delta_\xi(t-a)dt$$

$$= \lim_{\xi\to 0}\int_a^{a+\xi} e^{-st}\delta_\xi(t-a)dt = \lim_{\xi\to 0}\int_a^{a+\xi} e^{-st}\frac{1}{\xi}dt$$

$$= \lim_{\xi\to 0}\frac{1}{\xi}\left[\frac{e^{-st}}{-s}\right]_a^{a+\xi} = \lim_{\xi\to 0} -\frac{1}{\xi s}\left[e^{-s(a+\xi)} - e^{-as}\right]$$

$$= \lim_{\xi\to 0}\frac{1}{s\xi}\left[e^{-as} - e^{-as} - e^{-s\xi}\right]$$

$$= \frac{e^{-as}}{s}\lim_{\xi\to 0}\left[\frac{1-e^{-a\xi}}{s}\right]$$

$$= \frac{e^{-as}}{s}\lim_{\xi\to 0}\frac{se^{-s\xi}}{1} = \frac{e^{-as}}{s}s = e^{-as}$$

28. Find $L^{-1}\left[\frac{e^{-2s}}{s-3}\right]$

$$L^{-1}\left[e^{-2s}\frac{1}{s-3}\right] = L^{-1}\left[e^{-as}F(s)\right] = f(t-a)u(t-a)$$

$$F(s) = \frac{1}{s-3} \Rightarrow L^{-1}[F(s)] = L^{-1}\left[\frac{1}{s-3}\right] = e^{3t}$$

$$L^{-1}\left[e^{-2s}\frac{1}{s-3}\right] = e^{3(t-2)}u(t-2)$$

29. Find $L\left[\frac{\sin at}{t}\right]$. Hence show that $= \int_s^{\infty}\frac{\sin t}{t}dt = \frac{\pi}{2}$

$$L\left[\frac{\sin at}{t}\right] = \int_s^{\infty} L[\sin at]ds = \int_s^{\infty}\frac{a}{s^2+a^2}ds$$

$$= \left[a.\frac{1}{a}\tan^{-1}\left(\frac{s}{a}\right)\right]_s^{\infty} = \tan^{-1}(\infty) - \tan^{-1}\left(\frac{s}{a}\right) = \frac{\pi}{2} - \tan^{-1}\left(\frac{s}{a}\right)$$

$$= \cot^{-1}\left(\frac{s}{a}\right)(or)\tan^{-1}\left(\frac{a}{s}\right)$$

Put s = 0 and a = 1 we get

$$\int_s^{\infty}\frac{\sin t}{t}dt = \frac{\pi}{2}$$

30.Find $L^{-1}\left[\frac{1}{s(s-a)}\right]$

$$L^{-1}\left[\frac{1}{s(s-a)}\right] = \int_0^t L^{-1}[F(s)]dt = \int_0^t L^{-1}\left[\frac{1}{s(s-a)}\right]dt$$

$$= \int_0^t e^{at}dt = \left[\frac{e^{at}}{a}\right]_0^t = \frac{1}{a}[e^{at}-1]$$

31. Find $L^{-1}\left[\frac{s^2}{(s^2-a^2)^2}\right]$

$$L^{-1}\left[\frac{s^2}{(s^2-a^2)^2}\right]=L^{-1}\left[s.\frac{s}{(s^2-a^2)^2}\right]$$

$$=\frac{d}{dt}L^{-1}\left[\frac{s}{(s^2-a^2)^2}\right]=\frac{d}{dt}\left[\frac{t}{2a}\sinh at\right]$$

$$=\frac{1}{2a}[at\cosh at+\sinh at]$$

UNIT-V

LAPLACE TRANSFORMS

PART - C QUESTIONS

1. Using convolution theorem find $L^{-1}\left[\frac{1}{(s+1)\,(s+2)}\right]$.

2. Using convolution theorem find $L^{-1}\left[\frac{s^2}{(s^2+a^2)\,(s^2+b^2)}\right]$.

3. Find the inverse Laplace transform of the following function using convolution Theorem $\frac{1}{s^3(s+5)}$.

4. Using convolution theorem find $L^{-1}\left[\frac{2}{(s+1)(s^2+4)}\right]$.

5. Using Laplace transform method solve $y''-2y'+y=e^t$ given $y(0)=2$ and $y'(0)=1$.

6. Solve the differential equation using Laplace transform $y''+4y'+4y=e^{-t}$ given that $y(0)=0$ and $y'(0)=0$.

7. Solve by using Laplace transform $y''-3y'+2y=4$ given that $y(0)=2$, $y'(0)=3$.

8. Using Laplace transform method solve $y''+25y=10\cos 5t$ given $y(0)=2$ and $y'(0)=0$.

9. Using Laplace transform method solve $\frac{d^2y}{dt^2}+2\frac{dy}{dt}+2y=5\sin t$ given that $y(0)=y'(0)=0$.

10. Solve using Laplace transform, $y''+3y'+2y=e^{-t}$ given $y(0)=1$ and $y'(0)=0$.

11. Using Laplace transform method solve $y'' + 2y' - 3y = \sin t$ given that $y(0) = 0$ & $y'(0) = 0$

12. Using Laplace transform method solve $y'' + 6y' + 5y = e^{-2t}$ given that $y(0) = 0$ & $y'(0) = 1$.

13. Find the Laplace transforms of f(t) if $f(t) = e^{t}$, $0 < t < 2\pi$ & $f(t) = f(t + 2\pi)$.

14. Find the Laplace transform of the triangular wave function $f(t) = \begin{cases} t & 0 < t < 1 \\ 2 - t & 1 < t < 2 \end{cases}$ and $f(t) = f(t + 2)$.

15. Find the Laplace transforms of the periodic function $f(t) = \begin{cases} 1 \text{ for } 0 < t < a \\ -1 \text{ for } a < t < 2a \end{cases}$.

16. Find the Laplace transform of $f(t) = \begin{cases} \sin t & \text{when } 0 < t < \pi \\ 0 & \text{when } \pi < t < 2\pi \end{cases}$ and $f(t)$ is periodic with period 2π.

17. Find the Laplace transforms of rectangular wave function given by $f(t) = \begin{cases} A \text{ for } 0 < t < \frac{T}{2} \\ -A \text{ for } \frac{T}{2} < t < T \end{cases}$ and $f(t + T) = f(t)$.

18. Find the Laplace transform of the periodic function $f(t) = \begin{cases} t & 0 < t < \pi \\ 2\pi - t & \pi < t < 2\pi \end{cases}$.

19. Find the Laplace transform of the triangular wave function $f(t) = \begin{cases} t & 0 < t < b \\ 2b - t & b < t < 2b \end{cases}$.

20. Find the Laplace transforms of $f(t) = \begin{cases} -E \text{ for } 0 < t < \pi \\ E \text{ for } \pi < t < 2\pi \end{cases}$ and $f(t + 2\pi) = f(t)$.

21. Find the Laplace transform of $f(t) = \begin{cases} \sin \omega t \text{ when } 0 < t < \frac{\pi}{\omega} \\ 0 \text{ when } \frac{\pi}{\omega} < t < \frac{2\pi}{\omega} \end{cases}$ and $f(t)$ is periodic with period $\frac{2\pi}{\omega}$.