# ENGINEERING MATHEMATICS – II

## IMPORTANT QUESTIONS

## PART – C : UNIT – I : MULTIPLE INTEGRALS

1. Change the order of integration in $\int_0^1 \int_{x^2}^{2-x} xy \, dy \, dx$ and hence evaluate it.

2. Change the order of integration and evaluate $\int_0^4 \int_{x^2/4}^{2\sqrt{x}} dy \, dx$

3. Evaluate $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} \, dx \, dy$ by changing the order of integration.

4. Find the area included between the curves $y^2 = 4x$ and $x^2 = 4y$.

5. Find the volume of the Sphere $x^2+y^2+z^2 = a^2$ using triple integration.

6. Find the volume of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

## UNIT – II : VECTOR CALCULUS :

1. Find the angle between the normals to the Surface $xy = z^2$ at the points $(-2, -2, 2)$ and $(1, 9, -3)$

2. Show that $\vec{F} = (y^2 + 2xz^2)\vec{i} + (2xy - z)\vec{j} + (2x^2 z - y + 2z)\vec{k}$ is irrotational and hence find its Scalar potential.

3. Find the value of the constants a, b, c so that the vector $\vec{F} = (x+2y+az)\vec{i} + (bx-3y-z)\vec{j} + (4x+cy+2z)\vec{k}$ is irrotational.

4. Verify Gauss divergence theorem for $\vec{F} = 4xz\,\vec{i} - y^2\vec{j} + yz\,\vec{k}$ over the cube bounded by $x=0$, $x=1$, $y=0$, $y=1$ and $z=0$, $z=1$.

5. Verify Stoke's theorem for $\vec{F} = (y-z)\vec{i} + yz\,\vec{j} - xz\,\vec{k}$ where S is the surface bounded by the planes $x=0$, $x=1$, $y=0$, $y=1$, $z=0$, $z=1$ above the XOY plane.

6. Verify Stoke's theorem for $\vec{F} = (x^2-y^2)\vec{i} + 2xy\vec{j}$ taken around the rectangle bounded by the line $x=0$, $x=a$, $y=0$, $y=a$.

7. Verify Green's theorem in the plane to evaluate $\int_C (3x^2-8y^2)\,dx + (4y-6xy)\,dy$ where C is the boundary of the region defined by $x=0$, $y=0$ and $x+y=1$.

## UNIT III : COMPLEX DIFFERENTIATION

1. Find the analytic function f(z) whose real part is $u(x,y) = 3x^2y + 2x^2 - y^3 - 2y^2$

(2) Given that $u = \dfrac{\sin 2x}{\cosh 2y - \cos 2x}$, find the analytic function $f(z) = u + iv$.

(3) Find the analytic function whose real part is $e^{x}(x\cos y - y\sin y)$.

(4) If $f(z)$ is a regular function of $z$, prove that

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4\,|f'(z)|^2$$

(5) Find the bilinear transformation which maps the points $z = 0, -1, i$ onto the points $w = i, 0, \infty$.

(6) Find the image of the circle $|z-1| = 1$ in the complex plane under the mapping $w = \frac{1}{z}$.

(7) Find the image of the infinite strip (i) $0 < y < 1/2$ (ii) $\frac{1}{4} < y < \frac{1}{2}$ under the transformation $w = \frac{1}{z}$.

## UNIT - IV : COMPLEX INTEGRATION

(1) Find the Laurent's expansion of (i) $\dfrac{1}{(z+1)(z+3)}$ in the region $1 < |z| < 3$ (ii) $\dfrac{7z-2}{z(z+1)(z-2)}$ in the region $1 < |z+1| < 3$

(2) Find the Laurent's Series expansion of $f(z) = \dfrac{1}{(z-1)(z-2)}$ in the region (i) $|z| > 2$ (ii) $0 < |z-1| < 1$

③ Using Cauchy's integral formula, evaluate $\int_C \frac{z+4}{z^2+2z+5}\,dz$

where $c$ is the circle $|z+1-i| = 2$.

④ Using Cauchy's integral formula, evaluate $\int_C \frac{z\,dz}{(z-1)(z-2)}$

where $C$ is the circle $|z-2| = \frac{1}{2}$

⑤ Using Cauchy's residue theorem, evaluate

$\int_C \frac{z-1}{(z+1)^2(z-2)}\,dz$ where $c$ is $|z-i| = 2$.

⑥ Using Cauchy's residue theorem, evaluate

(i) $\int_C \frac{z}{(z-1)(z-2)^2}\,dz$, where $c$ is the circle $|z-2| = \frac{1}{2}$

(ii) $\int_C \frac{z-2}{z(z-1)}\,dz$, where $c$ is $|z| = 2$.

⑦ Evaluate $\int_C \frac{(\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-1)}\,dz$ where $c$ is the

circle $|z| = 3$ using (i) Cauchy's integral formula

(ii) Cauchy's residue theorem.

## UNIT – V : LAPLACE TRANSFORMS

① Find (i) $L\left(\frac{\cos at - \cos bt}{t}\right)$ (ii) $L(te^{-t}\cos t)$

(iii) $L\left(\frac{1-e^t}{t}\right)$

2. Verify initial and final value theorem for

$f(t) = 1 + e^{-t}(\sin t + \cos t)$

3. Find Laplace transform of

$$f(t) = \begin{cases} t, & 0 < t < a \\ 2a - t, & a < t < 2a \end{cases} \quad \text{with } f(t+2a) = f(t)$$

4. Find Laplace transform of

$$f(t) = \begin{cases} E, & 0 < t < \omega/2 \\ -E, & \omega/2 < t < \omega \end{cases} \quad \text{with } f(t+\omega) = f(t)$$

5. Applying convolution theorem, find

(i) $L^{-1}\left(\frac{s^2}{(s^2+a^2)^2}\right)$ (ii) $L^{-1}\left(\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right)$

6. Solve $y'' - 3y' - 4y = 2e^{-t}$, $y(0) = 1 = y'(0)$ using Laplace transforms.

7. Solve $y'' + 4y' + 3y = e^{-t}$, $y(0) = 1$ and $y'(0) = 0$ using Laplace transforms.