Bilinear transformation

$$\frac{(w-w_1)(w_2-w_3)}{(w-w_3)(w_2-w_1)} = \frac{(z-z_1)(z_2-z_3)}{(z-z_3)(z_2-z_1)}$$

Type I: one of the value of z-plane or w-plane is ∞.

(1) Find the bilinear transformation that maps the points $z = 0, 1, \infty$ into $w = i, 1, -i$ resp.

Soln:

$z_1 = 0$, $z_2 = 1$, $z_3 = \infty$

$w_1 = i$, $w_2 = 1$, $w_3 = -i$

$$\frac{(w-i)(1+i)}{(w+i)(1-i)} = \frac{(z-0)}{(1-0)} = \frac{z}{1}$$

$$\frac{(w-i)}{(w+i)}\frac{(1+i)}{(1-i)} \times \frac{1+i}{1-i} = z$$

$$\frac{(w-i)(1+2i-1)}{(w+i)(1+1)} = z$$

$$\frac{(w-i)\cancel{2}i}{\cancel{2}(w+i)} = z$$

$(w-i)i = z(w+i)$

$wi + 1 = zw + zi$

$wi - zw = zi - 1$

$w(i-z) = zi - 1$

$$\boxed{w = \frac{iz-1}{i-z}}$$

Type 2 : No Value is ∞

(1) Find the bilinear trans mapping the points $z = 1, i, -1$ into the points $w = 2, i, -2$ resp.

Soln:

$$\frac{(w-2)(i+2)}{(w+2)(i-2)} = \frac{(z-1)(i+1)}{(z+1)(i-1)}$$

$$\frac{(w-2)}{(w+2)} = \frac{(z-1)(i+1)(i-2)}{(z+1)(i-1)(i+2)}$$

$$= \frac{(z-1)(-3-i)}{(z+1)(i-3)}$$

$$\frac{w-2}{w+2} = +\frac{(z-1)(3+i)}{(z+1)(3-i)}$$

By Componendo - Dividendo rule, $\left[\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a+b}{a-b} = \frac{c+d}{c-d}\right]$

$$\frac{w-2+w+2}{w-2-w-2} = \frac{(z-1)(3+i)+(z+1)(3-i)}{(z-1)(3+i)-(z+1)(3-i)}$$

$$\frac{2w}{-4} = \frac{6z-2i}{2(iz-3)}$$

$$\boxed{w = \frac{-6z+2i}{iz-3}}$$

(2) $z = 1, i, -1$ onto $w = i, 0, -i$

Soln:

$$\boxed{w = \frac{1+iz}{1-iz}}$$

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