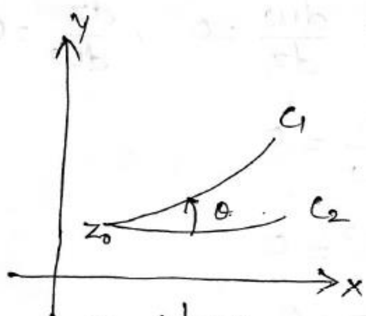
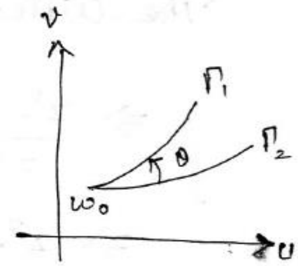
CONFORMAL MAPPING

Defn: A mapping $w = f(z)$ is said to be conformal at $z = z_0$ if it preserves the angle between any two curves through z_0 in z plane both in magnitude and direction.



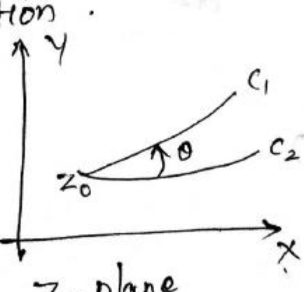
z plane



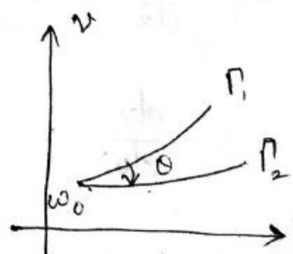
w plane

ISOGONAL MAPPING:

Defn: A mapping $w = f(z)$ is said to be isogonal at $z = z_0$ if it preserves the angle between any two curves through z_0 in z plane only in magnitude but not in direction.



z-plane



w-plane

REMARKS:

1. If $f(z)$ is analytic and $f'(z) \neq 0$ at each point then the mapping $w = f(z)$ is conformal.
2. The points at which $w = f(z)$ is not conformal (i.e) $f'(z) = 0$ are called critical points.
3. The critical points of $w = f(z)$ will occur at $\frac{dz}{dw} = 0$ and $\frac{dw}{dz} = 0$.

① Find the critical points of the transformation $w = z + 1/z$.

Soln:

$$\frac{dw}{dz} = 1 - \frac{1}{z^2} = \frac{z^2 - 1}{z^2}$$

$$\frac{dz}{dw} = \frac{z^2}{z^2 - 1}$$

The critical pts occur at $\frac{dw}{dz} = 0$, $\frac{dz}{dw} = 0$

$$\Rightarrow \frac{z^2-1}{z^2} = 0 \quad \text{and} \quad \frac{z^2}{z^2-1} = 0$$

$$\Rightarrow z^2 - 1 = 0 \quad \text{and} \quad z^2 = 0$$

$$\Rightarrow z = \pm 1 \quad \text{and} \quad z = 0$$

$\therefore$ $z = 0, 1, -1$ are the critical points.