CONTENT

- **Compton Effect**
- **Compton theory (derivation)**
- **Compton experimental verification**
- **Properties of Matter waves**
- **G.P Thomson experiment from Planck's theory**

SCATTERING OF X-RAYS

Two Kinds:

- Coherent scattering or classical scattering or **Thomson scattering**
- Incoherent scattering or **Compton scattering**

Coherent scattering:

- X rays are scattered without any **change in wavelength.**
- Obeys classical **electromagnetic theory**

Compton scattering:

- Scattered beam consists of **two wavelengths.**
- One is having same wavelength as the **incident beam**
- The other is having a slightly longer wavelength called **modified beam.**

COMPTON EFFECT

A beam of monochromatic **radiation** (x-rays, γ-rays) of high frequency fall on **a low atomic no. substance**(carbon, graphite),the beam is scattered into two components.

(i) **Modified radiation** – having lower frequency or larger wavelength

(ii) **Un Modified radiation** – having same frequency or wavelength

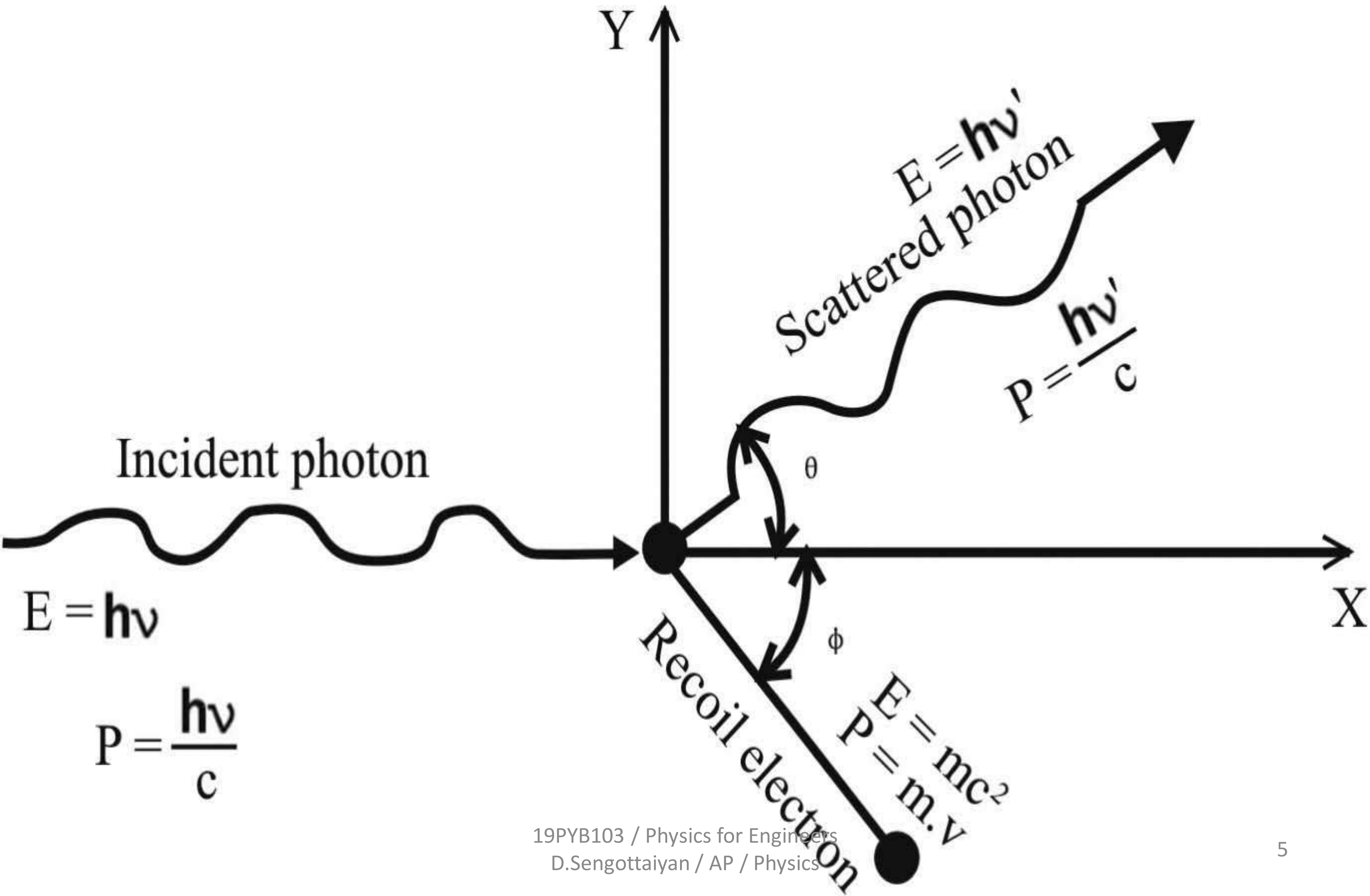
This change in wavelength of the scattered X rays is known as the Compton shift.

This effect is called **Compton Effect**.

THEORY OF COMPTON EFFECT

- Compton treated this scattering as the interaction between X ray and the matter as a particle collision between X ray photon and loosely bound electron in the matter.
- Consider an X ray photon of frequency ν striking an electron at rest.
- This Photon is scattered through an angle θ to x-axis.
- Let the frequency of the scattered photon be ν'.
- During collision the photon gives energy to the electron.
- This electron moves with a velocity V at an angle ϕ to x axis.

THEORY OF COMPTON EFFECT



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THEORY OF COMPTON EFFECT

Total Energy before collision:

Energy of the incident photon $= \mathbf{h\nu}$

Energy of the electron at rest $= m_0 C^2$

where m_0 is the rest mass of electron and C the velocity of light.

Therefore total energy before collision $= h\nu + m_0 C^2$

Total energy after collision:

energy of the scattered photon $= \mathbf{h\nu'}$

Energy of the Recoil electron $= \mathbf{mC^2}$

Therefore total energy after collision $= \mathbf{h\nu' + mC^2}$

Total energy before collision = Total energy after collision

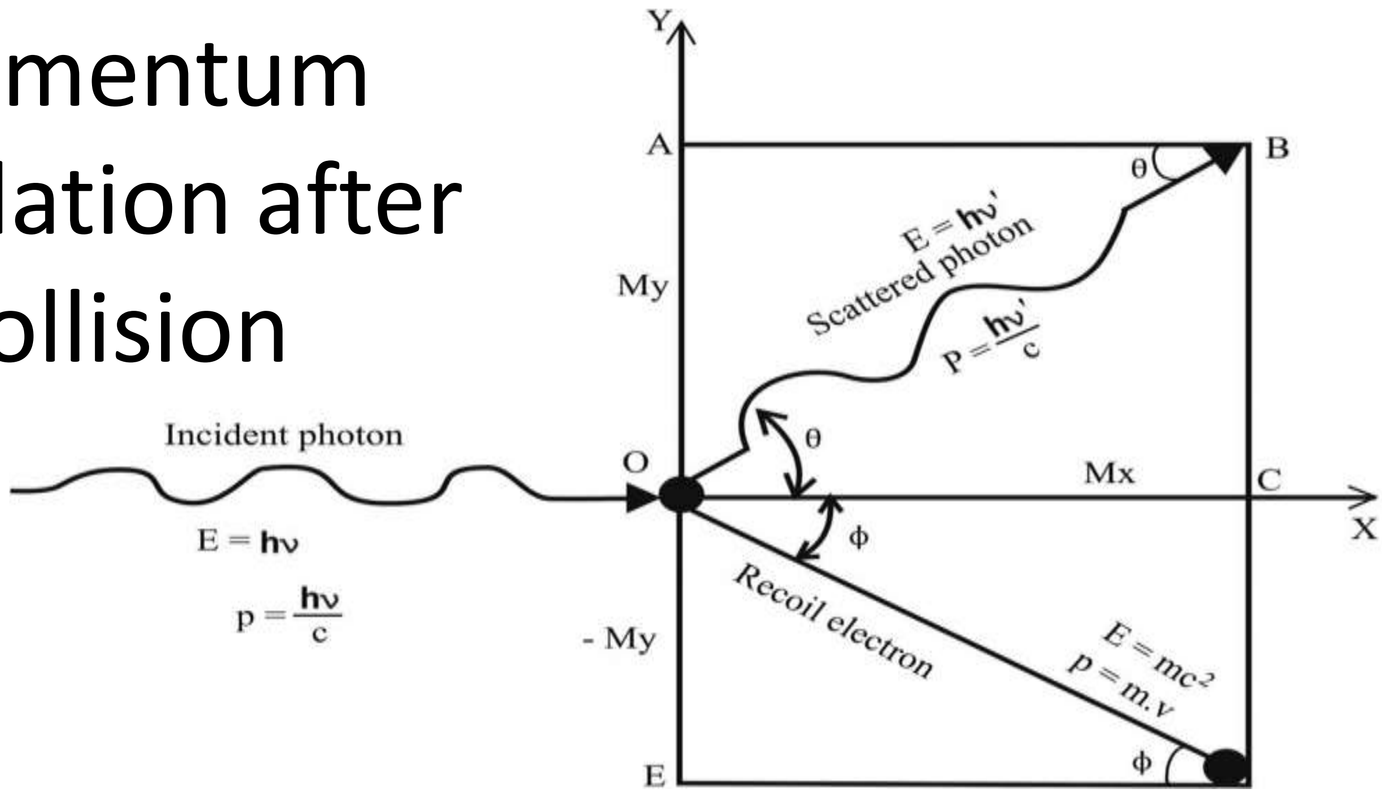
THEORY OF COMPTON EFFECT

By the law of conservation of energy

Total energy before collision = Total energy after collision

$$h\nu + m_0 C^2 = h\nu' + mC^2$$

Momentum calculation after collision



THEORY OF COMPTON EFFECT

Total momentum along x axis before collision

Momentum of incident photon along x axis $= \frac{h\nu}{C}$

Momentum of electron at rest along x axis = 0

Total momentum before collision along x axis $= \frac{h\nu}{C}$

Total momentum along x axis after collision

The momentum is resolved along x axis and y axis.

momentum of scatterd photon along x axis $= \frac{h\nu'}{C} \cos\theta$

momentum of recoil electron along x axis $= mV\cos\phi$

Total momentum after collision along X axis $= \frac{h\nu'}{C}\cos\theta + mV\cos\phi$

THEORY OF COMPTON EFFECT

, Applying the law of conservation of momentum

Momentum before collision = momentum after collision

$$\frac{h\nu}{C} = \frac{h\nu'}{C}Cos\theta + mVCos\phi$$

$$\frac{h\nu}{C} - \frac{h\nu'}{C}Cos\theta = mVCos\phi$$

$$h\nu - h\nu' Cos\theta = mVCCos\phi......(2)$$

Total momentum along y axis before collision

Initial momentum of photon along y axis =0
Initial momentum of electron along y axis = 0
Total momentum before collision along y axis= 0

THEORY OF COMPTON EFFECT

Total momentum along y axis after collision

momentum of scattered photon along y axis $= \frac{h\nu}{C} sin\theta$

momentum of recoil electron along y axis $= -mvsin\phi$

(along the negative Y direction)

Total momentum after collision along y axis $= \frac{h\nu}{C} sin\theta - mvsin\phi$

Momentum before collision = momentum after collision

$$0 = \frac{h\nu'}{C} Sin\theta - mVSin\phi$$

$$h\nu' Sin\theta = mVCSin\phi .....(3)$$

THEORY OF COMPTON EFFECT

Squaring (2) and (3) and adding,

$$m^2V^2C^2(Cos^2\phi + Sin^2\phi) = h^2(\nu - \nu' Cos\,\theta)^2 + h^2\nu'^2\,Sin^2\theta$$

$$m^2V^2C^2(Cos^2\phi + Sin^2\phi) = h^2(\nu^2 - 2\nu\nu' Cos\,\theta + \nu'^2\,Cos^2\theta) \;+ h^2\nu'^2\,Sin^2\theta$$

$$m^2V^2C^2(Cos^2\phi + Sin^2\phi) = h^2\nu^2 - 2h^2\nu\nu' Cos\,\theta + h^2\nu'^2\,(Sin^2\theta + Cos^2\theta)$$

$$m^2V^2C^2 = h^2\nu^2 - 2h^2\nu\nu' Cos\,\theta + h^2\nu'^2 \,.....(4)$$

Squaring (1) ,

$$m^2C^4 = [h(\nu - \nu') + m_0C^2]^2$$

$$m^2C^4 = h^2(\nu - \nu')^2 + 2h(\nu - \nu')m_0C^2 + m_0^{\,2}C^4$$

$$m^2C^4 = h^2(\nu^2 - 2\nu\nu' + \nu'^2) + 2h(\nu - \nu')m_0C^2 + m_0^{\,2}C^4 \,.....(5)$$

THEORY OF COMPTON EFFECT

Eqn (5)-(4)

$$m^2C^4 - m^2C^2V^2 = h^2(\nu^2 - 2\nu\nu' + \nu'^2) + 2h(\nu - \nu')m_0C^2 + m_0{}^2C^4 - h^2(\nu^2 - 2\nu\nu' Cos\theta + \nu'^2)$$

$$m^2C^2(C^2 - V^2) = -2h^2\nu\nu' + 2h(\nu - \nu')m_0C^2 + m_0{}^2C^4 + 2h^2\nu\nu' Cos\theta$$

$$m^2C^2(C^2 - V^2) = 2h(\nu - \nu')m_0C^2 + m_0{}^2C^4 - 2h^2\nu\nu'(1 - Cos\theta)..(6)$$

From the theory the variation of mass with velocity is given by

$$m = \frac{m_0}{\sqrt{1 - \frac{V^2}{C^2}}} \ldots(7)$$

Squaring (7)

$$m^2 = \frac{m_0{}^2}{1 - \frac{v^2}{C^2}} = \frac{m_0{}^2}{\frac{C^2 - v^2}{C^2}} = \frac{m_0{}^2C^2}{C^2 - v^2}$$

$$m^2(C^2 - V^2) = m_0^2C^2 \ldots(8)$$

THEORY OF COMPTON EFFECT

Substituting (8) in (6)

$$m_0{}^2 C^4 = -2h^2 \nu\nu'(1 - \cos\theta) + 2h(\nu - \nu')m_0 C^2 + m_0^2 C^4$$

$$2h^2 \nu\nu'(1 - \cos\theta) = 2h(\nu - \nu')m_0 C^2$$

$$\frac{(\nu - \nu')}{(\nu\nu')} = \frac{h}{m_0 C^2}(1 - \cos\theta)$$

$$\frac{\nu}{\nu\nu'} - \frac{\nu'}{\nu\nu'} = \frac{h}{m_0 C^2}(1 - \cos\theta) : \frac{1}{\nu'} - \frac{1}{\nu} = \frac{h}{m_0 C^2}(1 - \cos\theta)$$

Multiplying by C on both the sides,

$$\frac{C}{\nu'} - \frac{C}{\nu} = \frac{h}{m_0 C}(1 - \cos\theta) \; : \; \lambda' - \lambda = \frac{h}{m_0 C}(1 - \cos\theta)$$

Therefore the change in wavelength is given by $d\lambda = \frac{h}{m_0 C}(1 - Cos\theta)$

- The change in wavelength **dλ does not depend** on the
  - (i) wavelength of the **incident photon**
  - (ii) Nature of the **scattering material**
- The change in wavelength **dλ depends** only on the **scattering angle.**

THEORY OF COMPTON EFFECT

Case (1) When **θ=0** then,

$$d\lambda = \frac{h}{m_0 C}(1 - 1) = 0 \qquad d\lambda = 0$$

Case (2) When **θ=90º** then,

$$d\lambda = \frac{h}{m_0 C}(1 - 0) = \frac{h}{m_0 C}$$

Substituting the values for h,m_0 and C

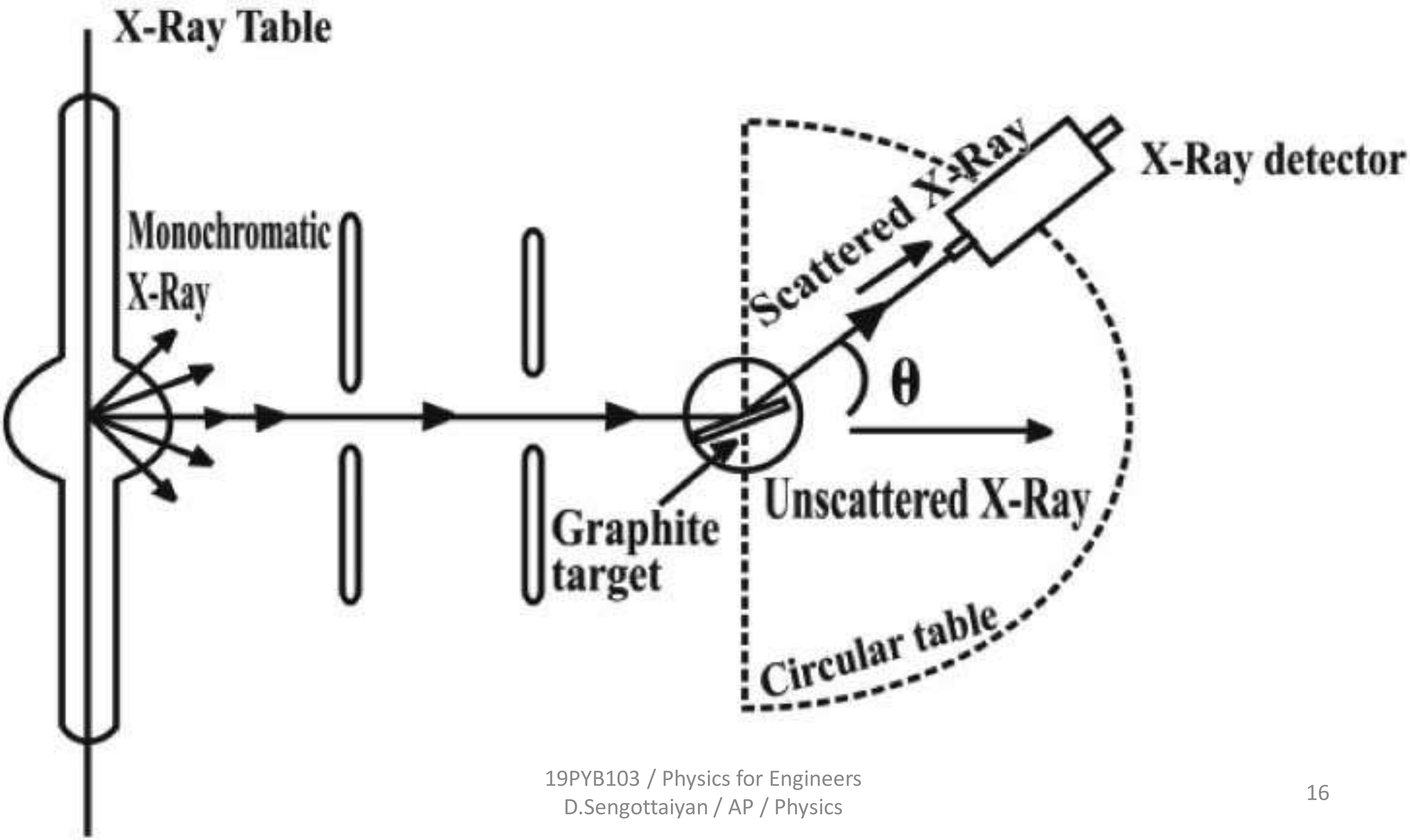
$$d\lambda = \frac{h}{m_0 C} = \frac{6.63x10^{-34}}{9.11x10^{-31}x3x10^8} = 0.0243\ A^0$$

Case(3) When **θ=180º** then,

$$d\lambda = \frac{h}{m_0 C}(1 - (-1)) \quad = 0.048Å$$

The change in wavelength is maximum at 180^0

EXPERIMENTAL VERIFICATION OF COMPTON EFFECT



EXPERIMENTAL VERIFICATION OF COMPTON EFFECT

1. The experimental set up is as shown in the fig.

2. A beam of mono chromatic X ray beam is allowed to fall on the **scattering material.**

3. The scattered beam is received by a **Bragg spectrometer**.

4. The **intensity of the scattered beam** is measured for various angles of scattering.

5. A graph is plotted between the **intensity and the wavelength.**

EXPERIMENTAL VERIFICATION OF COMPTON EFFECT

6. Two peaks were found.

7. One belongs to unmodified and the other belongs to the **modified beam.**

8. The difference between the two peaks gives the **shift in wavelength.**

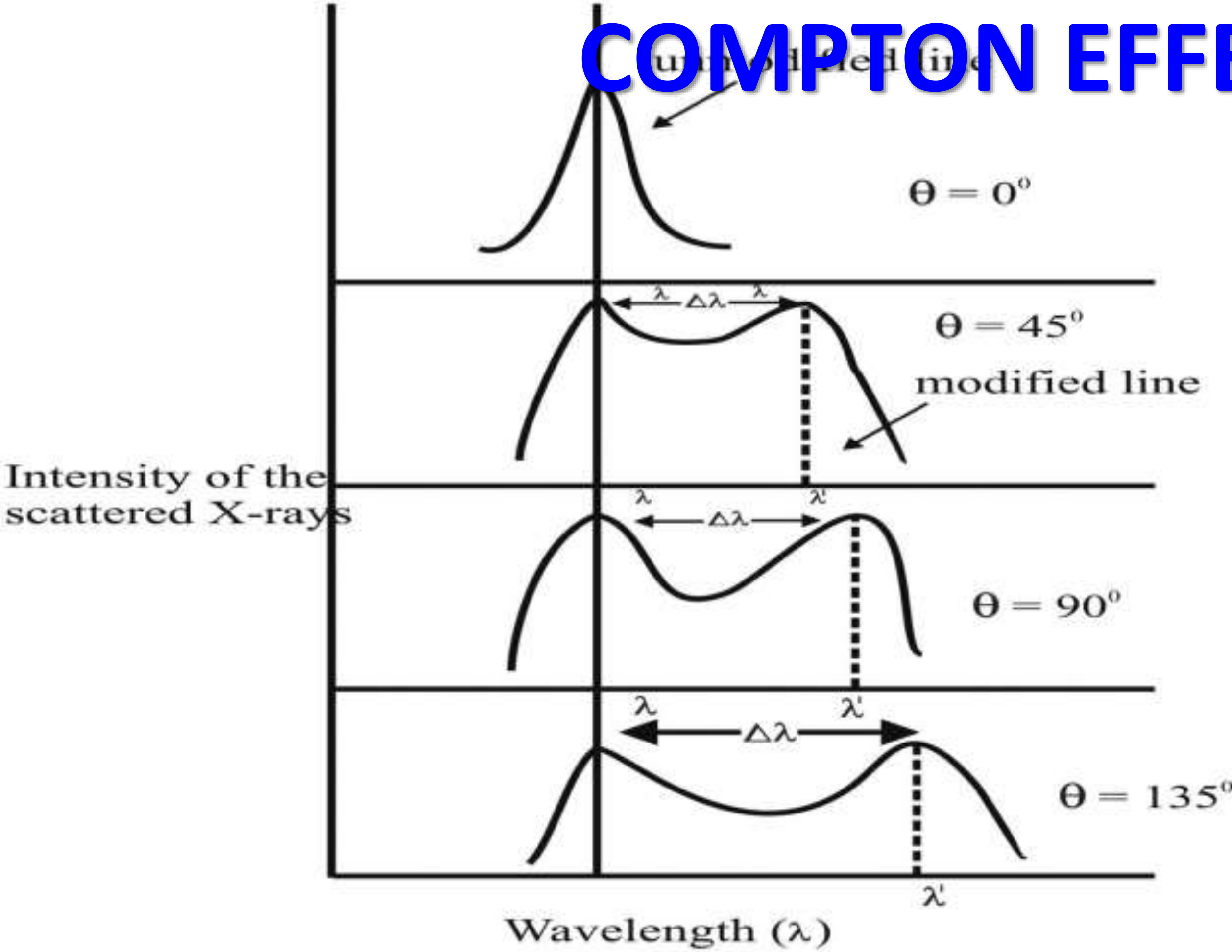
9. When the scattering angle is increased the shift also gets increased in accordance with

$$d\lambda = \frac{h}{m_0 C}(1 - \cos\theta) \ldots\ldots 12$$

10. The experimental values were found to be in good agreement with that found by the formula.

11. Compton's results at the scattering angles 0, 45 ∘, 90 ∘ and 135∘, are shown in the following Figure.

EXPERIMENTAL VERIFICATION OF COMPTON EFFECT



The wavelength of the scattered *X -rays becomes* longer as the scattering angle increases.

Compton shift ***Δλ is zero*** *when* ***ϑ = 0***

Compton shift is ***maximum*** when ***ϑ = 135°.***