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#### DEPARTMENT OF MATHEMATICS UNIT -V LAPLACE TRANSFORM

PERIODIC FUNCTIONS : A junct fit is said to be periodie if f (E+T) = f(E) for all values of t and for certain valuer of T. The smallest value of T for which of (t+T)= Z(t) for all t is called the period of the func. gO! The funct. Shith cost are periodic functions both having period 271.  $sht = sh(t + 2\pi) = sh(t + 4\pi) = ...$ consider the func. fit)= St if o<t<2 and fit+47=fit; (4-t if 2<t<4 ...f(t) is a periodic func. with period 4. IT & periodic functions: Letfit) be a periodic function with period T. Then  $L = \frac{1}{1 - e^{-ST}} \int^{T} e^{-St} f(t) dt.$ 





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() Find LT of Z(E) = (21), 05153& Z(E+3)=Z(E) Soln: f(E) & a portable func with period 3. (i) T=3 WITT  $L \Gamma_{f(t)} = \frac{1}{1 - e^{-st}} \int e^{-st} f(t) dt$  $= \frac{1}{1 - e^{-3S}} \int_{e^{-St}}^{3} \left(\frac{2t}{3}\right) dt = \frac{1}{1 - e^{-3S}} \left(\frac{2}{3}\right) \int_{e^{-St}}^{St} dt$  $= \frac{1}{1-e^{-3s} \left(\frac{2}{3}\right)} \left[\frac{1}{1-s} - \frac{1}{s} - \frac{1}{s}\right]^{3}$  $= \frac{1}{1 - e^{-3S}} \left(\frac{2}{3}\right) \left[\frac{3e^{-3S}}{-S} - \frac{e^{-3S}}{s^2} + \frac{1}{S^2}\right]$  $= \frac{1}{1 - e^{-3S}} \left(\frac{2}{3}\right) \left[\frac{1 - e^{-3S}}{5^2} - \frac{3e^{-3S}}{5^2}\right]$ (2) Find the LT of fit & fit) = et oxt < 2TT and fit)=f(t+2TT Solni Z(E) % a periodie function with period 211 (1) T=211  $L[f(t)] = \frac{1}{1-e^{-2\pi s}} \int_{e^{-st}}^{2\pi} e^{-st} dt$  $= \frac{1}{1 - e^{-2\pi i s}} \int_{0}^{2\pi} e^{(1-s)t} dt = \frac{1}{1 - e^{-2\pi i s}} \frac{e^{(1-s)t}}{(1-s)} \int_{0}^{2\pi i s} \frac{e^{(1-s)t}}{(1-s)} dt = \frac{1}{1 - e^{-2\pi i s}} \frac{e^{(1-s)t}}{(1-s)} \frac{e^{(1-s)t}}{(1-s$  $= e^{2\pi(1-s)}$  $(1-s)(1-e^{-2\pi s})$ 

19MAB102/ Integral Calculus & Laplace Transform

S.Sindhuja/AP/Maths/SNSCT PAGE -2 of 3





(An Autonomous Institution) Coimbatore – 35

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3) Find LT of f(t)= ft, oct~1 such that 12-t, 1<t<2 f(t+2)=f(t) Som: f(t) is a periodic func with period 2 wis T=2.  $L [f(t)] = \frac{1}{1 - e^{-2s}} \int e^{-st} f(t) dt$ =  $\frac{1}{1-e^{-2s}}\int e^{-st}t dt + \int e^{-st}(z-t) dt$  $= \frac{1}{1 - e^{-2S}} \int_{-S}^{S} \frac{e^{-S}}{8^2} + \frac{1}{8^2} + \frac{1}{8^2} + \frac{e^{-2S}}{8^2} + \frac{e^{-S}}{8^2} - \frac{e^{-S}}{8^2} \Big\}^{\frac{1}{2}}$  $= \frac{1}{1 - e^{-2S}} \int \frac{1 - e^{-S}}{S^2} - \frac{e^{-S}}{S} + \frac{e^{-2S}}{e^2} + \frac{e^{-S}}{S} \int \frac{1}{S} \frac{1 - e^{-S}}{S} + \frac{$  $= \frac{1}{1 - e^{-2S}} \int_{S^2}^{1} \frac{1}{S^2} \left[ e^{-2S} - 2e^{-S} + 1 \right]_{S^2}^{2}$  $= \frac{(1-e^{-s})^2}{s^2(1-e^{-s})} \xrightarrow{(1-e^{-s})^2}_{s \in [1-(e^{-s})^2]} \xrightarrow{(1-e^{-s})^2}_{s \in (1-e^{-s})^2}$ ) Find LT of the periodic func.  $f(t) = \begin{cases} 1, 0 < t < a \\ -1, a < t < 2a \end{cases}$ f(t+2a) = f(t).  $\frac{1}{1-e^{-Sq}} = \frac{(1-e^{-Sq})^2}{c(1-e^{-2qs})}$ 

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UNIT -V LAPLACE TRANSFORM