

Young's Modulus(Y):

It is defined as the ratio between the longitudinal stress to the longitudinal strain, within the elastic limits.

$$\text{Young's Modulus (Y)} = \frac{\text{longitudinal stress}}{\text{longitudinal strain}}$$

Uniform Bending

Consider a beam supported by two knife edges A and B. Length between A and B is 'l'. Let equal weights (W), be added to either end of the beam C and D.

Let the distance CA and BD = a. Due to load applied the elevation 'x' produced from F to E. Let W be the reaction produced at the points A and B which acts vertically upwards.

From the fig

The external bending moment about P, written as

$$M_p = Wa$$

We know the internal bending moment = $\frac{YI_g}{R}$

On comparing (1) and (2)

$$Wa = \frac{YI_g}{R}$$

Here it is found that the elevation 'x' forms an arc of the circle of radius R

From ΔAFO

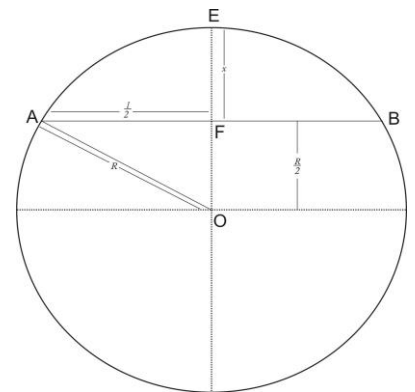
$$OA^2 = AF^2 + FO^2$$

Since $OF = FE$, therefore we can write $OA^2 = AF^2 + FE^2$

$$\text{(or)} \quad AF^2 = OA^2 - FE^2$$

$$AF^2 = FE \left[\frac{OA^2}{FE} - FE \right]$$

Here, $AF = \frac{l}{2}$, $FE = x = \frac{R}{2}$; $OA = R$



$$\left(\frac{l}{2}\right)^2 = x \left[\frac{R^2}{R/2} - x \right]$$

$$\left(\frac{l^2}{4}\right) = 2xR - x^2$$

If elevation x is small

$$\left(\frac{l^2}{4}\right) = 2xR$$

$$x = \frac{l^2}{8R} \quad (\text{or}) \quad R = \frac{l^2}{8x}$$

$$(\text{or}) \quad W_a = \frac{YI_g}{l^2/8x} \quad (\text{or}) \quad W_a = \frac{8YI_g x}{l^2}$$

On Rearranging

The elevation of point 'E' above 'A' is $x = \frac{W_a l^2}{8YI_g}$