Euler Graph and Hamilton Graph:

Eulerian path:

A path of a graph G is called an Eulerian path, if it contains each edge of the graph exactly once.

Eulerian circuit (or) Eulerian cycle:

A circuit or cycle of a graph G is called an Eulerian circuit or cycle, if it includes each edge of G exactly once and starting and ending points are same.

Eulerian graph:

Any graph containing an Eulerian circuit or cycle is called an Eulerian graph.

Note: A connected graph is Euler iff each of its vertices is of even degree.

Hamiltonian Graph:

Hamiltonian path:

A path of a graph G is called a Hamiltonian path, if it includes each vertex of G exactly once

Hamiltonian circuit or cycle

A circuit of a graph G is called a Hamiltonian circuit (cycle) if it includes each vertex of G exactly once, except the starting & ending vertices.

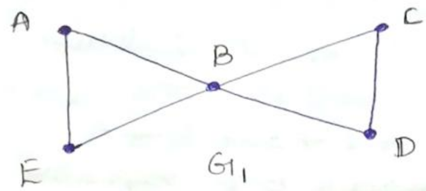
Hamiltonian graph:

Any graph containing a Hamiltonian circuit or cycle is called a Hamiltonian graph.

1. Give an example of a graph which is
   i). Eulerian but not Hamiltonian
   ii). Hamiltonian but not Eulerian
   iii). Both Eulerian and Hamiltonian
   iv). Non Eulerian and Non Hamiltonian.

Soln.

i). Eulerian graph but not Hamiltonian



G_1

Since G_1 contains an Eulerian circuit.

ie, $A \to B \to C \to D \to B \to E \to A$ (All the edges occur exactly once)

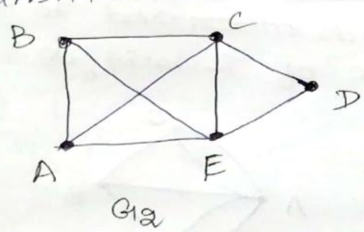
The degree of all vertices of G_1 is even.

$\therefore$ G_1 is Eulerian

As the vertex B is repeated twice, G_1 is not a Hamiltonian graph.

$\therefore$ G_1 is Eulerian but not Hamiltonian.

ii). Hamiltonian but not Eulerian:



G_2

Since G_2 contains Hamiltonian cycle $A \to B \to C \to D \to E \to A$ (All vertices occur exactly once)

$\therefore$ G_2 is Hamiltonian.

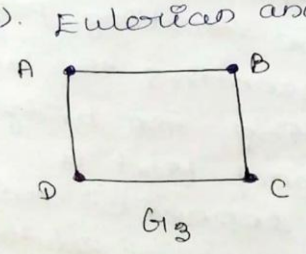
Since the degree of A and B is not an even

$\Rightarrow$ G_2 is not Eulerian graph.

$\Rightarrow$ G_2 is not Eulerian.

$\therefore$ G_2 is Hamiltonian but not Eulerian.

iii). Eulerian and Hamiltonian



G_3

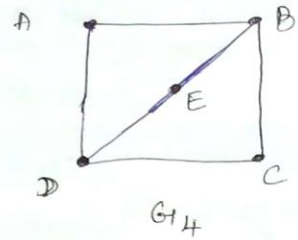
In G_3 contains the cycle

$A \to B \to C \to D \to A$

Since the cycle contains all the edges exactly once & all the vertices exactly once. And also all the vertices are of even degree.

$\therefore$ G_3 is both Eulerian and Hamiltonian.

iv). Non-Eulerian and Non-Hamiltonian:



Since deg B and deg D are not even nos

$\therefore$ G_4 is not an Eulerian graph.

Since G_4 contains the cycle

$A \to B \to C \to D \to E \to B \to A$.

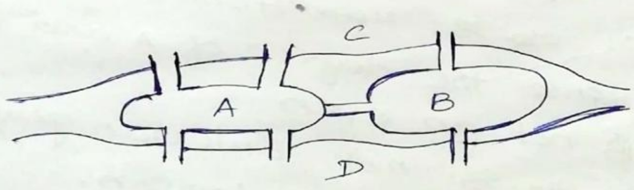
As the vertex B is repeated twice, the graph is not Hamiltonian graph

$\therefore$ G_4 is Non-Eulerian and non-Hamiltonian.

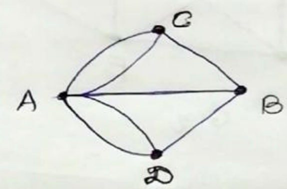
2]. Explain Konigsberg bridge problem. Represent the problem by mean of graph. Does the problem have a solution?

Soln.

There are two islands A and B formed by a river. They are connected to each other and to the river banks C and D by means of 7 bridges.



Konigsberg Bridge problem



Graphical Representation

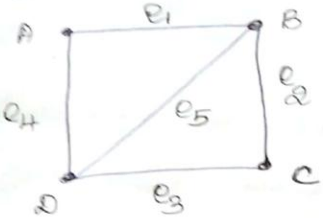
The problem is to start from any one of the 4 land areas A, B, C, D walk across each bridge exactly once and return to the starting point.

In this graph, vertices representing the land areas and the edges representing the bridges.

In the above graph, we cannot find a Eulerian circuit ie, the edge repeated twice

Hence Konigsberg bridge Problem has no solution.

3]. Find all possible Eulerian path and Eulerian cycle of the gvn. graph. Is it Euler graph?



Possible Euler paths:

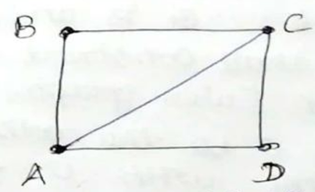
i). B → D → C → B → A → D
ii). B → C → D → A → B → D
iii). B → A → D → C → B → D
iv). D → C → B → A → D → B
v). D → B → C → D → A → B
vi). D → A → B → C → D → B

Euler cycles:

i). B → D → C → B → A → D → B
ii). B → C → D → A → B → D → B
iii) B → A → D → C → B → D → B
iv). D → C → B → A → D → B → D
v). D → B → C → D → A → B → D
vi). D → A → B → C → D → B → D

Gvn. graph is not a Euler graph.

4]. Find Hamiltonian path and Hamiltonian cycle, if it exist in each of the graphs given below. Also identify which graph is Hamiltonian.



Possible Hamiltonian paths:

i). A → B → C → D
ii) A → D → C → B
iii) B → C → D → A
iv). B → A → D → A
v). C → D → A → B
vi) C → B → A → D
vii) D → A → B → C
viii). D → C → B → A

Possible Hamiltonian cycles:

i). A → B → C → D → A
ii). A → D → C → B → A
iii) B → C → D → A → B
iv). B → A → D → A → B
v). C → D → A → B → C
vi) C → B → A → D → C
vii). D → A → B → C → D
viii). D → C → B → A → D

Gvn. graph is Hamiltonian graph.