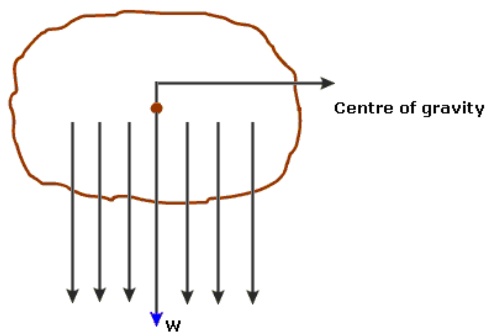
UNIT-III PROPERTIES OF SURFACES AND SOLIDS

Syllabus

Determination of Areas and Volumes – First moment of area and the Centroid of sections – Rectangle, circle, triangle from integration – T section, I section, Angle section, Hollow section by using standard formula – second and product moments of plane area – Rectangle, triangle, circle from integration – T section, I section, Angle section, Hollow section by using standard formula – Parallel axis theorem and perpendicular axis theorem – Polar moment of inertia – Principal moments of inertia of plane areas – Principal axes of inertia - Mass moment of inertia – Derivation of mass moment of inertia for rectangular section, prism, sphere from first principle – Relation to area moments of inertia.

1. **Define Centre of Gravity**

   Centre of gravity is defined as an imaginary point at which the entire weight of the body is assumed to act.



$$x = \frac{W_1 X_1 + W_2 X_2 + W_3 X_3 + ...}{W_1 + W_2 + W_3 + ...}$$

$$y = \frac{W_1 Y_1 + W_2 Y_2 + W_3 Y_3 + ...}{W_1 + W_2 + W_3 + ...}$$

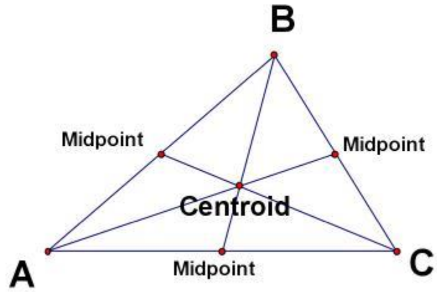
Where,

W_1 , W_2 ….are weight of different components of composite solid.

X_1 , X_2 ….and Y_1 , Y_2 .....distances of their respective C.G from the reference axis in X and Y direction

2. **Define Centroid**

   Centroid is defined as the point at which the entire area of the body is assumed to be concentrated.



$$x = \frac{A_1 X_1 + A_2 X_2 + A_3 X_3 + ...}{A_1 + A_2 + A_3 + ...} \; or \; \frac{V_1 X_1 + V_2 X_2 + V_3 X_3 + ...}{V_1 + V_2 + V_3 + ...} \; or \; \frac{L_1 X_1 + L_2 X_2 + L_3 X_3 + ...}{L_1 + L_2 + L_3 + ...}$$

$$y = \frac{A_1 Y_1 + A_2 Y_2 + A_3 Y_3 + ...}{A_1 + A_2 + A_3 + ...} \; or \; \frac{V_1 Y_1 + V_2 Y_2 + V_3 Y_3 + ...}{V_1 + V_2 + V_3 + ...} \; or \; \frac{L_1 Y_1 + L_2 Y_2 + L_3 Y_3 + ...}{L_1 + L_2 + L_3 + ...}$$

Where,

A_1 , A_2 ….are cross sectional area of different components of composite solid.

V_1 , V_2 …. are volume of different components of a body.

L_1 , L_2 …. are length of different parts of lines.

X_1 , X_2 ….and Y_1 , Y_2 .....distances of their respective C.G from the reference axis in X and Y direction

3. **When will the centroid and centre of gravity coincide?**
When gravitational field is uniform and parallel. i.e. if the body is homogenous or density of the body is uniform throughout the body then the centre of gravity and centroid are coincide.

4. **Define moment of inertia of a body**
Moment of inertia (I) about an axis is the algebraic sum of the products of the elements of mass and the square of the distance of the respective element of mass from the axis.

$$\mathbf{I = \sum m_1 r_1^2}$$

5. **Define centroid axis**
Centroidal axis is defined as that axis which passes through the centre of gravity of the body or through the centroid of an area

6. **Define radius of gyration**
Radius of gyration of a body is defined as the distance from axis of reference to where the whole mass of a body is assumed to be concentrated so as to not alter the moment of inertia about the given axis.Radius of gyration $k = \sqrt{I / A}$, where I = Moment of inertia, A = Total area of the plane

7. **Differentiate centre of gravity and centroid**

S.No	Centre of gravity(C.G)	Centroid(G)
1	Centre of gravity is defined as an imaginary point at which the entire weight of the body is assumed to act	Centroid is defined as the point at which the entire area of the body is assumed to be concentrated
2	It is a physical property of a body	It is a geometric property of body

3	It has both area and mass	It has only area and no mass
4	It refers to three dimensional body	It refers to one, two dimensional body
4	It is applied to solids.E.g. wire, rod	It is applied to area. E.g. line, area, volume

8. State pappus and guldinus theorems

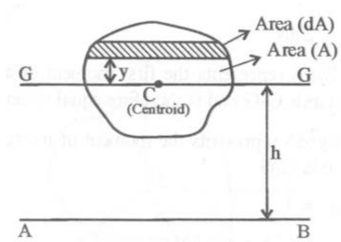
Theorem –I: The area of a surface of revolution is equal to the product of length of the generating curve and the distance travelled by the centroid of the curved line while the surface is being generated

Theorem-II: The volume of a body of revolution is equal to the product of the generating area and the distance travelled by the centroid of the area while the body is being generated.

9. State parallel axis theorem

Moment of inertia of a plan area which is parallel to centroidal axis about an axis is equal to the sum of the moment of inertia about centroidal axis and the product of area and square of the distance between the two parallel axes.

$$\mathbf{I_{AB} = I_G + Ah^2}$$



Where I_{AB} = moment of inertia about parallel axis x-x

I_G=moment of inertia about its centroidal axis G-G

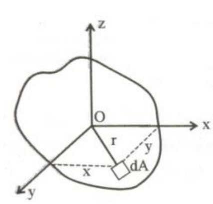
A= Area of the body

h= distance between the axes GG and XX

10. **State perpendicular axis theorem or Polar moment of inertia**.

Moment of inertia of an area about two mutually perpendicular axes passing through its centroid is equal to the sum of the moment of inertia about two mutually perpendicular axes passing through centroid and in the plane of body by considering the body of area.

$$\mathbf{I_{ZZ} = I_{XX} + I_{YY}}$$



11. What are principal axes?

Principal axes are the set of mutually perpendicular axes where the product of inertia is zero.

12. What are principal moment of inertia?

Moment of inertia of a section about principal axes are called as principal moment of inertia.

13. Define polar moment of inertia

Polar moment of inertia is defined as the moment of inertia of the lamina or plane about the axis perpendicular to the plane of the section. It is denoted by Ixx or J

14. Define mass moment of inertia

The mass moment of inertia about an axis is the product of elemental mass and the square of the distance between the mass centre of the elemental mass and the axis.

Mass moment of inertia =$\int x^2\, dm$

15. What is section modulus?

The modulus of section of a figure is the quantity obtained by dividing the moment of inertia of the figure about its C.G by the distance of the extreme fibre from the centroidal axis

16. How many centre of gravity a body has?

Only one.

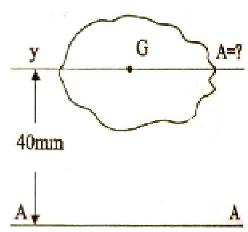
.

17. **Write the formula for principal axes?**

Principal axes are a set of mutually perpendicular axes where the product of inertia is zero. The position of the axes are given by

$$\tan 2\theta = \frac{2I_{xy}}{I_{yy} - I_{xx}}$$

18. **Given that M.I I_{xx} =500 cm^4 and M.I about an axis AA which is parallel to centroidal xx axis at a distance of 40 mm is 900 cm^4. Determine the area?**



<u>Given data</u>

$I_{xx} = I_G = 500\ cm^4$

$I_{AA} = 900\ cm^4$

Height h = 40mm = 4cm

<u>Solution</u>

Moment of inertia $I_{AA} = I_G + Ah^2$

$900 = 500 + A \times 40^2$

Area (A) = 25 cm^2

19. **Write the formula for principal moment of inertia?**

Moment of inertia of a section about principal axes are called as principal moment of inertia and is given by the equation

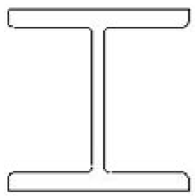
$$I_{max}, I_{min} = \frac{I_{xx}+I_{yy}}{2} \pm \sqrt{\left(\frac{I_{xx}-I_{yy}}{2}\right)^2 + I_{xy}^2}$$

20. What is meant by moment of inertia?

The property which gives the measure of resistance to bending in the case of plane area or plates is known as moment of inertia of the area

21.What is axis of symmetry?

The axis about which similar configuration exist with respect to shape, size and weight on either side is known as axis of symmentry



Column

I section is symmetrical about both x-axis and y –axis

22.What is axis of revolution?

The fixed axis about which a plane curve or plane area is rotated is known as axis of revolution

22. Define centre of mass.

Center of mass is the point where the entire mass of a body is assumed to be concentrated. The position of centre of mass depends upon the shape and density of the body. The mass centre may (or may not) necessarily lie within the boundary of the body. Each body has only one mass centre for all positions of the body. The mass centre for a composite solids made up of different materials is given by,

$$x = \frac{\rho_1 V_1 X_1 + \rho_2 V_2 X_2 + \rho_3 V_3 X_3 + ...}{\rho_1 V_1 + \rho_2 V_2 + \rho_3 V_3 + ...}$$

$$y = \frac{\rho_1 V_1 Y_1 + \rho_2 V_2 Y_2 + \rho_3 V_3 Y_3 + ...}{\rho_1 V_1 + \rho_2 V_2 + \rho_3 V_3 + ...}$$

Where,

ρ, V , are density and volume of different components of composite solid.

X_1 , X_2 ….and Y_1 , Y_2 .....distances of their respective C.G from the reference axis in X and Y direction

23. Using the theorems of Pappus, determine the surface area and volume of a right circular cone, with radius of base r and height h.

Surface area = Length of the curve x Distance traveled by the centroid during revolution

$= l \times 2\pi y = l \times 2\pi (r/2)$

$A_S = \pi rl$

Volume = area of the generating area × distance traveled by the centroid during one revolution.

$= \frac{1}{2} hr \times 2 \times \pi \times r/3$

$V = \pi r^2 h/3$

24. Write the expression to find the co ordinates of centroid by integration method?

for plane figure $\bar{x} = \frac{\int x_s dA}{\int dA}$; $\bar{y} = \frac{\int y_s dA}{\int dA}$

for solid figure , $\bar{x} = \frac{\int x_m dm}{\int dm}$; $\bar{y} = \frac{\int y_m dm}{\int dm}$

25. Under what condition do the following coincide? a). Center of mass and \ Centre of gravity b). center of gravity and centroid of area .

A. centre of gravity coincides with the centre of mass if the gravitational forces are taken to be uniform and parallel.

B. If a lamina (plate) has uniform thickness and is homogeneous, its centre of gravity coincides with the centroid if the area.

26. What is axis of revolution and axis of symmetry?

Axis of revolution

The fixed axis about which a plane curve(may be an arc, straight line) or plane area is rotated is known as axis of revolution.

Axis of symmetry

The axis about which similar configuration exist with respect to shape , size an weight on either side is known as axis o symmetry. It may be horizontal, vertical or inclined.

27. State the salient properties of principal axes.

1) If the given area has an axis of symmetry passing through a point, then the axis must be principal axis need not necessarily be an axis of symmetry.
2) The properties of principal axes hold good for any point located inside or outside the given area.
3. If the given point coincides with the centroid of an area, the two principal axes of the area about the centroid are known as the principal centroidal axes of the area.

28. **What do you understand from principle moments of inertia**

The perpendicular axes about which "Product of inertia" is zero are called 'principal axes' and the moment of inertia with respect to these principal axes are called as "Principal moments of inertia"

The maximum moment of inertia about principal axes is called Major principal moment of inertia.

The minimum moment of inertia about principal axes is called Major principal moment of inertia.

29. **Define mass moment of inertia of a solid body**

The moment of inertia of solid figure is called mass moment of inertia. For plane figure, masses are assumed to be negligible and hence area of plane figures are taken to find the moment of inertia, whereas for solid figures masses are considered.

Mass moment of inertia of any solid = Thickness x Density x Area Moment of Inertia of the solid about the same axis

$(I_{xx})_{mass} = (I_{XX})_{area} \times M$

Where,

M- Mass of the solid
$(I_{xx})_{mass}$ –Mass moment of inertia
$(I_{XX})_{area}$ - Area moment of inertia

30. A rectangle has a width of 2m and height of 3m. Find its product of inertia about a set of coordinate axes passing through its top left corner and parallel to its sides.

$$I_{xy} = xydxdy$$

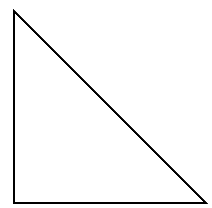
$$\text{here } I_{xy} = \int_0^2 \int_{-3}^0 xydxdy = \frac{-2^2 \times 3^2}{4} = -9 \text{ mm}^4$$

31 Find the product of inertia of a right angled triangle with respect to x and y axes are shown in fig

$$I_{xy} = \frac{b^2 h^2}{24}$$

$$= \frac{20^2 \times 40^2}{24}$$

$$= 26.667 \times 10^3 mm^4$$



32 semi circular area having radius 100 mm is located in the xy plane such that its. Diametrical edge coincides with the y axis. Determine the x- coordinate of its centroid.

$$x = \frac{4r}{3\pi}$$

$$x = \frac{4 \times 100}{3\pi}$$

$$= 42.44 \text{ mm}$$

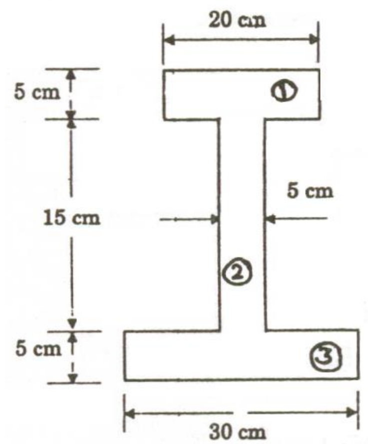
33 State the relationship between the second moment of area and mass moment of inertia for a thin uniform plate.

Mass moment of inertia of thin plate about any axis

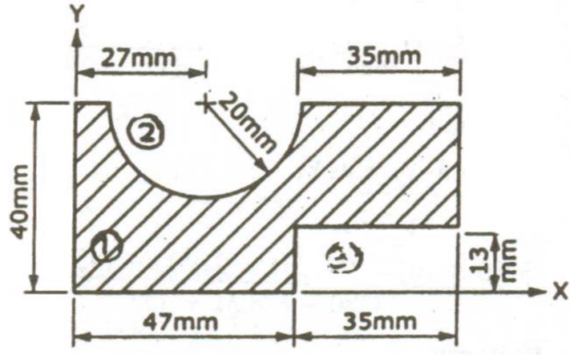
= thickness × density × second moment of area of the plate about the same axis.

PART-B

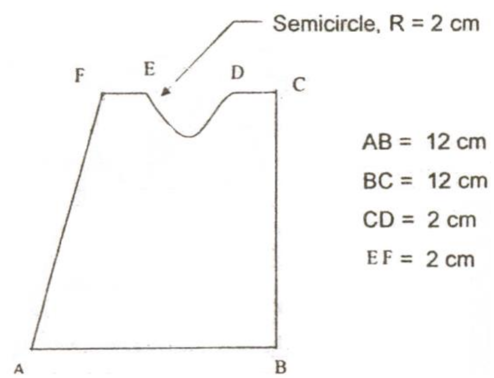
1. Determine the centroid of the cross-sectional area of an unequal I –section shown below.



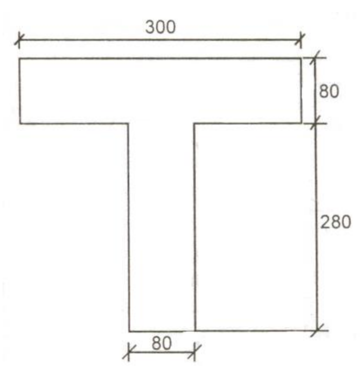
2. Find the centroid of the shaded area shown below.



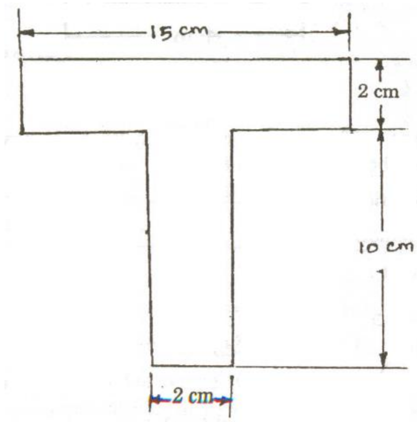
3. Find the moment of inertia of a plane area as shown in figure about its centroidal X-axis



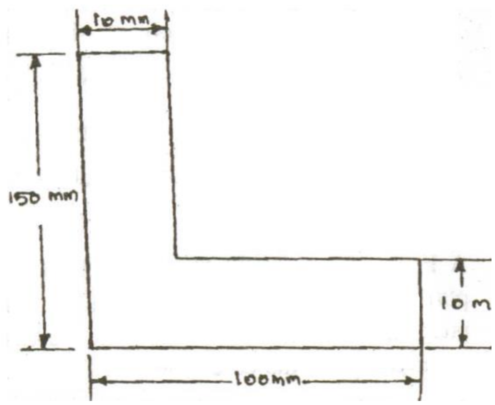
4. Find the polar moment of inertia of a T-section shown in figure about an axis passing through its centroid. Also find the radius of gyration with respect to the polar axis (dimensions are in mm)



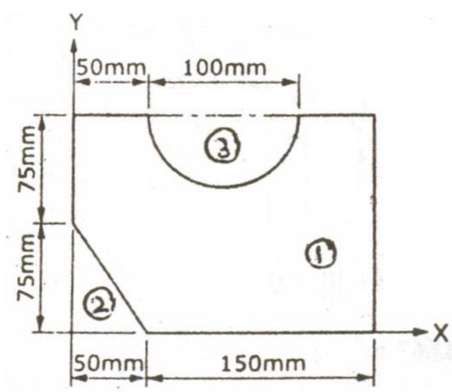
5. Find the centre of gravity of the T-section shown in figure



6. Find the principal moment of inertia of the L section about the centroidal axis shown in the figure.



7. For the plane area shown in the figure, determine the area, moment of inertia and radius of gyration about the x- axis



8 Determine the moment of inertia of beam, cross –sectional area shown below about the x and y centroidal axes.