



SNS COLLEGE OF TECHNOLOGY
(AN AUTONOMOUS INSTITUTION)
COIMBATORE - 35
DEPARTMENT OF MATHEMATICS



Converse, Contrapositive and Inverse proposition:

Defn:

If $P \rightarrow Q$, then

$Q \rightarrow P$ is called its converse

$\neg Q \rightarrow \neg P$ is called its contrapositive

$\neg P \rightarrow \neg Q$ is called its Inverse.

Remarks:

i. The conditional proposition and its contrapositive are logically equivalent. i.e., $(P \rightarrow Q) \Leftrightarrow (\neg Q \rightarrow \neg P)$

ii. The conditional proposition and its converse are not logically equivalent. i.e., $(P \rightarrow Q) \not\Leftrightarrow (Q \rightarrow P)$

Example:

1. Obtain converse, contrapositive and inverse for the statement "Team India wins whenever Dhoni is a captain"

Now, P : Dhoni is a captain

Q : Team India wins

$P \rightarrow Q$: If Dhoni is a captain, then Team India wins. (conditional)

$Q \rightarrow P$: If team India wins then dhoni is a captain. (converse)

$\neg Q \rightarrow \neg P$: If the crops does team India does not wins then dhoni is not a captain. (contrapositive)

$\neg P \rightarrow \neg Q$: If Dhoni is not a captain then team India does not wins.

2. Obtain "If it rains then the crops will grow."

P : It rains

Q : The crops will grow.

$P \rightarrow Q$: If it rains then the crops will grow

$Q \rightarrow P$: If the crops will grow then it rains

$\neg Q \rightarrow \neg P$: If the crops will not grow then it does not rains

$\neg P \rightarrow \neg Q$: If it does not rains then the crops will not grow.

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Other connectives:

(i) NAND \rightarrow a combination of NOT & AND
denoted by \uparrow

(ii) NOR \rightarrow a combination of NOT & OR
denoted by \downarrow

which is defined as

$$P \uparrow Q = \neg(P \wedge Q) \quad \text{and} \quad P \downarrow Q = \neg(P \vee Q)$$

Normal forms:

The statement written in the standard form
in terms of \vee , \wedge and \neg then it is called the normal forms.

Note: (i) conjunction (\wedge) is denoted as product.

(ii) disjunction (\vee) is denoted as sum.

Elementary product:

A prod. of the variables and their negations
in a formula is called an elementary product.

Eg: P , $\neg P \wedge Q$, $\neg Q \wedge P$, $P \wedge \neg P$, $Q \wedge \neg P$

Elementary sum:

A sum of the variables and their negations
in a formula is called an elementary sum.

Eg: P , $\neg P \vee Q$, $\neg Q \vee P$, $P \vee \neg P$, $Q \vee \neg P$

Disjunctive Normal form (DNF)

A statement formula which is equivalent
to a given formula and which consists of a sum of
elementary products is called a disjunctive normal
form of the given formula.

$$\text{DNF} = (\text{Elementary product}) \vee (\text{Elementary product}) \vee \dots \vee (\text{Elementary product})$$

Conjunctive Normal form:

A statement formula which is equivalent to a
given formula and which consists of a product of
elementary sum is called a conjunctive normal form.

$$\text{CNF} = (\text{Elementary sum}) \wedge (\text{Elementary sum}) \wedge \dots \wedge (\text{Elementary sum})$$

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Obtain the DNF and CNF of the formula

$$P \rightarrow [(P \rightarrow Q) \wedge \neg (\neg Q \vee TP)]$$

DNF:

$$P \rightarrow [(P \rightarrow Q) \wedge \neg (\neg Q \vee TP)]$$

$$\Leftrightarrow TP \vee [(P \rightarrow Q) \wedge \neg (\neg Q \vee TP)] \quad \text{material implication law}$$

$$\Leftrightarrow TP \vee [(TP \vee Q) \wedge \neg (\neg Q \vee TP)] \quad \text{material implication law}$$

$$\Leftrightarrow TP \vee [(TP \vee Q) \wedge (Q \wedge P)] \quad \text{De Morgan's law}$$

$$\Leftrightarrow TP \vee [(TP \wedge (Q \wedge P)) \vee (Q \wedge (Q \wedge P))] \quad \text{distributive law}$$

$$\Leftrightarrow TP \vee [TP \wedge (Q \wedge P)] \vee [(Q \wedge Q) \wedge P] \quad \text{Associative law}$$

$$\Leftrightarrow TP \vee [TP \wedge (Q \wedge P)] \vee [Q \wedge P] \quad \text{Idempotent law}$$

CNF:

$$P \rightarrow [(P \rightarrow Q) \wedge \neg (\neg Q \vee TP)]$$

$$\Leftrightarrow TP \vee [(TP \vee Q) \wedge \neg (\neg Q \vee TP)] \quad \text{material implication law}$$

$$\Leftrightarrow TP \vee [(TP \vee Q) \wedge (Q \wedge P)] \quad \text{De Morgan's law}$$

$$\Leftrightarrow [TP \vee (TP \vee Q)] \wedge [TP \vee (Q \wedge P)] \quad \text{distributive law}$$

$$\Leftrightarrow [TP \vee Q] \wedge [TP \vee (Q \wedge P)] \quad \text{Idempotent law}$$

$$\Leftrightarrow (TP \vee Q) \wedge [(TP \vee Q) \wedge (TP \vee P)] \quad \text{distributive law}$$

$$\Leftrightarrow (TP \vee Q) \wedge (TP \vee Q) \wedge (TP \vee P)$$

$$\Leftrightarrow (TP \vee Q) \wedge (TP \vee P)$$

Obtain a DNF of $P \wedge (P \rightarrow Q)$

$$\text{Now } P \wedge (P \rightarrow Q) \Leftrightarrow P \wedge (TP \vee Q)$$

$$\Leftrightarrow (P \wedge TP) \vee (P \wedge Q) \quad \text{distributive law}$$

Since the given statement formula is written in terms of sum of elementary products.