4) $f(z) = \dfrac{1}{z-2}$ $\quad z = 1$

$f(z) = \dfrac{1}{z-2}$ $\quad f(1) = -1$

$f'(z) = \dfrac{-1}{(z-2)^2}$ $\quad f'(1) = -1$

$f''(z) = \dfrac{2}{(z-2)^3}$ $\quad f''(1) = \dfrac{2}{-1} = -2$

$f'''(z) = \dfrac{-6}{(z-2)^4}$ $\quad f'''(1) = -6$

$$= -1 + \frac{(z-1)}{1}(-1) + \frac{(z-1)^2}{2}(-2) + \frac{(z-1)^3}{6}(-6)$$

$$= -1 - (z-1) - (z-1)^2 - (z-1)^3$$

Laurent's series : * 14 mark

1) Expand $f(z) = \dfrac{7z-2}{z(z-2)(z+1)}$ in laurent's theo.

If i) $|z| < 2$

ii) $|z| > 3$

iii) $2 < |z| < 3$

iv) $1 < |z+1| < 3$

$$\frac{7z-2}{z(z-2)(z+1)} = \frac{A}{z} + \frac{B}{z-2} + \frac{C}{z+1}$$

$$\frac{7z-2}{z(z-2)(z+1)} = \frac{A(z-2)(z+1) + Bz(z+1) + Cz(z-2)}{z(z-2)(z+1)}$$

$$7z - 2 = A(z-2)(z+1) + Bz(z+1) + Cz(z-2)$$

Put $z=0$

$0-2=A(0-2)(0+1)$

$-2=-2A$

$\boxed{A=1}$

Put $z=2$

$7(2)-2=B\,2(2+1)$

$12=6B$

$\boxed{B=2}$

Put $z=-1$

$7(-1)-2=C(-1)(-1-$

$-9=3C$

$\boxed{C=-3}$

$$\frac{7z-2}{z(z-2)(z+1)}=\frac{1}{z}+\frac{2}{z-2}-\frac{3}{z+1}\rightarrow (1)$$

i) $|z|<2 \Rightarrow \frac{|z|}{2}<1$

$$(1)\Rightarrow \frac{7z-2}{z(z-2)(z+1)}=\frac{1}{z}+\frac{2}{2\left(\frac{z}{2}-1\right)}-\frac{3}{1+z}$$

$$=\frac{1}{z}-\left(1-\frac{z}{2}\right)^{-1}-3(1+z)^{-1}$$

$$=\frac{1}{z}-\left(1+\frac{z}{2}+\left(\frac{z}{2}\right)^3+\cdots\right)-3\left(1-z+z^2+z^3+\cdots\right.$$

ii) $|z|>3 \Rightarrow 3<|z| \Rightarrow \frac{3}{|z|}<1$

$$(1)\Rightarrow \frac{7z-2}{z(z-2)(z+1)}=\frac{1}{z}+\frac{2}{z-2}-\frac{3}{z+1}$$

$$=\frac{1}{z}+\frac{2}{z\left(1-\frac{2}{z}\right)}-\frac{3}{z\left(1+\frac{1}{z}\right)}$$

$$=\frac{1}{z}+\frac{2}{z}\left(1-\frac{2}{z}\right)^{-1}-\frac{3}{z}\left(1+\frac{1}{z}\right)^{-1}$$

$$=\frac{1}{z}+\frac{2}{z}\left(1+\frac{2}{z}+\left(\frac{2}{z}\right)^2+\cdots\right)-\frac{3}{z}\left(1-\frac{1}{z}+\left(\frac{1}{z}\right)^2-\left(\frac{1}{z}\right)^3+\cdots\right)$$

iii) $2<|z|<3 \Rightarrow 2<|z|$ & $|z|<3$

$\frac{2}{|z|}<1$ $\qquad$ $\frac{|z|}{3}<1$

$$(1)\ \frac{7z-2}{z(z-2)(z+1)}=\frac{1}{z}+\frac{2}{z-2}-\frac{3}{z+1}$$