3) using cauchy's integral formula find $\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz$ where c is a circle $x^2+y^2=9$

$$\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz \quad , \quad x^2+y^2 = 9$$

$$|z| = |x+iy|$$

$$|z| = \sqrt{x^2+y^2}$$

$$|z|^2 = x^2+y^2$$

$$|z|^2 = 9$$

$$z = 3$$

$$\frac{1}{(z-1)(z-2)} = \frac{A}{z-1} + \frac{B}{z-2}$$

$$\frac{1}{(z-1)\cancel{(z-2)}} = \frac{A(z-2)+B(z-1)}{(z-1)\cancel{(z-2)}}$$

$$1 = A(z-2) + B(z-1)$$

Put $z=1$

$$1 = A(1-2) + 0$$

$$1 = -A$$

$$\boxed{A = -1}$$

Put $z=2$

$$1 = 0 + B(2-1)$$

$$\boxed{B = 1}$$

$$\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz = \int -\frac{(\sin \pi z^2 + \cos \pi z^2)}{z-1} dz + \int \frac{\sin \pi z^2 + \cos \pi z^2}{z-2}$$

$$= 2\pi i f(1) + 2\pi i f(2)$$

$$= 2\pi i[-(-1)] + 2\pi i[1]$$

$$= 2\pi i + 2\pi i$$

$$= 4\pi i.$$

4) Evaluate $\int_C \frac{e^z}{(z-1)^3} dz$, where c is the circle $|z-1| = 3/2$

Sol:

$f(z) = e^z$, $a=1$

$f'(z) = e^z$

$f''(z) = e^z$

sub $a=1$ in c : $|z-1| = 3/2$

$|1-1| = 3/2$

$0 < 3/2$

$a=1$ lies inside C.

By cauchy's theorem,

$$\int_C \frac{f(z)}{(z-a)^3} = 2\pi i \frac{f''(a)}{2!}$$

$$\therefore \int_C \frac{e^z}{(z-1)^3} dz = 2\pi i \cdot \frac{f''(1)}{2}$$

$$= \frac{2\pi i \cdot e}{2}$$

$$= \pi i e$$,,.

5) If F(z) is analytic inside a circle. c with Centre $z=a$ then $f(z)$ can be expressed as

$$f(z) = f(a) + \frac{(z-a)}{1!} f'(a) + \frac{(z-a)^2}{2!} f''(a) + \frac{(z-a)^3}{3!} f'''(a)$$

which is convergent at every point inside c this is called taylor series. $f(z)$ at $z=a$