Note:

1) Tayler series of $f(z)$ at $z = 0$ is called maclaurin series.

2) $(1-x)^{-1} = 1 + x + x^2 + x^3 + \dots$

$(1+x)^{-1} = 1 - x + x^2 - x^3 + \dots$

$(1+x)^{-2} = 1 - 2x + 3x^2 - 4x^3 + \dots$

$(1-x)^{-2} = 1 + 2x + 3x^2 + 4x^3 + \dots$

1) expand $f(z) = \log(1+z)$ as tayler series about the point $z = 0$

Derivative	Derivative at $z=0$
$f(z) = \log(1+z)$	$f(0) = \log(1+0) = 0$
$f'(z) = \frac{1}{1+z}$	$f'(0) = 1$
$f''(z) = \frac{-1}{(1+z)^2}$	$f''(0) = -1$
$f'''(z) = \frac{2}{(1+z)^3}$	$f'''(0) = 2$

$$= f(0) + \frac{(z-a)}{1!} f'(0) + \frac{(z-a)^2}{2!} f''(0) + \frac{(z-a)^3}{3!} f'''(0)$$

$$= 0 + z(1) + \frac{z^2}{2}(-1) + \frac{z^3}{\cancel{6}_3} \times \cancel{2}$$

$$= z - \frac{z^2}{2} + \frac{z^3}{3} + \dots$$

2) Expand $f(z) = e^z$ as tayler series $z = 0$

Derivative	Derivative of $z=0$
$f(z) = e^z$	$f(0) = 1$

$f'(z) = e^z$ $\quad\quad f'(0) = 1$

$f''(z) = e^z$ $\quad\quad f''(0) = 1$

$f'''(z) = e^z$ $\quad\quad f'''(0) = 1$

$$f(z) = f(0) + \frac{(z)}{1!} f'(0) + \frac{z^2}{2!} f''(0) + \frac{z^3}{3!} f'''(0) + \dots$$

$$= 1 + z(1) + (1)\frac{z^2}{2} + \frac{z^3}{6} + \dots$$

$$= 1 + z + \frac{z^2}{2} + \frac{z^3}{6} + \dots$$

2) Expand $f(z) = \cos z$ about the point $z = \pi/3$

Derivative	Derivative of $z = \pi/3$
$f(z) = \cos z$	$f(\pi/3) = \cos \pi/3 = 1/2$
$f'(z) = -\sin z$	$f'(\pi/3) = -\sin \pi/3 = -\frac{\sqrt{3}}{2}$
$f''(z) = -\cos z$	$f''(\pi/3) = -\cos \pi/3 = -1/2$
$f'''(z) = \sin z$	$f'''(\pi/3) = \sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$

$$f(z) = f(\pi/3) + \frac{(z - \pi/3)}{1!} f'(\pi/3) + \frac{(z-\pi/3)^2}{2!} f''(\pi/3) + \frac{(z - \pi/3)^3}{3!} f'''(\pi/3) + \dots$$

$$= \frac{1}{2} + (z - \pi/3)\left(-\frac{\sqrt{3}}{2}\right) + \frac{(z-\pi/3)^2}{2}\left(\frac{-1}{2}\right) + \frac{(z-\pi/3)^3}{6}\frac{\sqrt{3}}{2}$$

$$= \frac{1}{2} - \frac{\sqrt{3}}{2}\left(z - \frac{\pi}{3}\right) - \frac{\left(z - \frac{\pi}{3}\right)^2}{2}\left(\frac{-1}{2}\right) + \frac{(z - \pi/3)^3}{6}\frac{\sqrt{3}}{2} + \dots$$