3-40

A pipe-line carrying water has average height of irregularities projecting from the surface of the boundary of the pipe as 0.15 mm. What type of boundary is it? The shear stress developed is 4.9 N/m². The kinematic viscosity of water is 0.1 stroke.

Given

Average height of irregularities $k = 0.15\,mm$

$= 0.15 \times 10^{-3}\,m$

Shear stress developed $\tau_0 = 4.9\,N/m^2$

Kinematic viscosity $\nu = 0.01$ stokes

$= 0.01\,cm^2/s$

$= 0.01 \times 10^{-4}\,m^2/s$

Density of water $\rho = 1000\,kg/m^3$

Shear velocity ① $u_* = \sqrt{\tau_0/\rho}$

$= \sqrt{\frac{4.9}{1000}}$

$= \sqrt{0.0049}$

$u_* = 0.07\,m/s$

② Roughness Reynold number $= \frac{u \cdot k}{\nu}$

$= \frac{0.07 \times 0.15 \times 10^{-3}}{0.01 \times 10^{-4}}$

$= 10.5$

Since $\frac{u \cdot k}{\nu}$ lies between 4 and 100 and hence pipe surface behaves as in transition.

3-41

A Smooth pipe of diameter 80mm and 800m long Carries water at the rate of 0.480 m³/min Calculate the loss of head, wall Shearing stress, Centre line Velocity, Velocity and Shear stress at 30mm from pipe wall. Also Calculate the thickness of Laminar Sub-Layer Take kinematic viscosity of water as 0.015 stokes.

Take the value of Co-efficient of friction 'f' from the relation given as

$$f = \frac{0.0791}{(R_e)^{1/4}} \qquad R_e = \text{Reynolds number.}$$

Given:

Dia of Smooth pipe $d = 80mm = 0.08m$

Length of pipe $L = 800m$

Discharge $Q = 0.048\ m^3/min$

$$= \frac{0.48}{60}$$

$$Q = 0.008\ m^3/sec$$

[Stokes = cm²/s]

Kinematic Viscosity $\nu = 0.015$ Stokes

$$= 0.015 \times 10^{-4}\ m^2/s$$

Density of water $\rho = 1000\ kg/m^3$

Mean velocity $V = \frac{Q}{Area}$

① ⇒ ① Calculate head lost
② wall shearing stress

To find:
Thickness of laminar sub layer. $= \frac{0.008}{\frac{\pi}{4}(0.08)^2}$

$$V = 1.591 \text{ m/s}$$

∴ Reynolds number $R_e = \frac{V \times d}{\nu} \Leftarrow \frac{\rho V d}{\rho \nu}$

$$= \frac{1.591 \times 0.08}{0.015 \times 10^{-4}}$$

$$R_e = 8.485 \times 10^4$$

As the Reynolds number is more than 4000 the flow is turbulent

Now the value of 'f' is given $f = \frac{0.0791}{R_e^{1/4}}$ ← given in problem.

$$f = \frac{0.0791}{(8.485 \times 10^4)^{1/4}}$$

$$f = 0.004636$$

(i) Head lost is given by equation $h_f = \frac{4fLV^2}{d\,2g}$

$$= \frac{4 \times 0.004636 \times 800 \times 1.591^2}{0.08 \times 2 \times 9.81}$$

$$h_f = 23.42 \text{ m.}$$

(ii) Wall shearing stress τ_0 is given by equation $\tau_0 = \frac{f \rho V^2}{2}$

$$\tau_0 = 0.004636 \times \frac{1000}{2} \times (1.591)^2$$

$$\boxed{\tau_0 = 5.866\ N/m^2}$$

(iii) Centre-line velocity u_{max} for smooth pipe is given by equation $\frac{u}{u_*} = 5.75 \log_{10} \frac{u \cdot y}{\nu} \times 5.55$

where u_* is shear velocity and $= \sqrt{\frac{\tau_0}{\rho}} = \sqrt{\frac{5.866}{1000}}$

$$= 0.0765\ m/s$$

The velocity will be maximum when $y = \frac{d}{2} = \frac{0.08}{2} =$

$$= 0.04\ m.$$

Hence at $y = 0.04\ m$, $u = u_{max}$ Substituting these values in (i) we get

$$\frac{u_{max}}{0.0765} = 5.75 \log_{10} \frac{0.0765 \times 0.04}{0.015 \times 10^{-4}} + 5.55$$

$$= 5.75 \log_{10} 2040 + 5.55$$

$$= 5.75 \times 3.309 + 5.55$$

$$= 19.03 + 5.55$$

$$= 24.58$$

$$U_{max} = 0.0765 \times 24.58 = 1.88\ m/s$$

(iv) The shear stress τ at any point is given by

$$\tau = -\frac{\partial p}{\partial x}\frac{r}{2} \quad \text{—— (A)}$$

where r = distance from centre of pipe and hence shear stress at pipe wall where $r = R$ is

$$\tau_0 = -\frac{\partial p}{\partial x}\frac{R}{2} \quad \text{—— (B)}$$

Dividing equation (A) by equation (B) we get

$$\frac{\tau}{\tau_0} = \frac{r}{R}$$

Shear stress $\tau = \frac{\tau_0 r}{R}$

A point 30mm from pipe wall is having $r = 4 - 3 = 1\,cm = 0.01\,m$

$$\tau \text{ at } (r = 0.01\,m) = \frac{\tau_0 \times 0.01}{0.04}$$

$$= \frac{5.866}{4}$$

$$= 1.4665\ N/m^2$$

Velocity at a point 3 cm from pipe wall means $y = 3\,cm = 0.03\,m$

and is given by equation as $\frac{u}{u_*} = 5.75 \log_{10}\frac{u_* y}{\nu} + 5.55$

where $u_* = 0.0765$ $y = 0.03$

$$\frac{u}{0.0765} = 5.75 \log_{10} \frac{0.0765 \times 0.03}{0.015 \times 10^{-4}} + 5.55$$

$$= 5.75 \log_{10} 1530 + 5.55$$

$$= 23.86$$

$$u = 0.0765 \times 23.86$$

$$= 1.825 \text{ m/s}$$

(v) Thickness of Laminar Sub-Layer is given by

$$\delta' = \frac{11.6 \times V}{u_*}$$

$$= \frac{11.6 \times 0.015 \times 10^{-4}}{0.0765}$$

$$= 2.274 \times 10^{-4} \text{ m}$$

$$= 2.274 \times 10^{-2} \text{ cm}$$

$$\boxed{\delta' = 0.02274 \text{ cm.}}$$

where $u = 1.5$ m/s at $y = (R - r) = 0.05 - 0.03$

$u = 0.02$ m

$$\frac{2 - 1.5}{u_*} = 5.75 \log_{10} \frac{0.05}{0.02}$$

$$= 2.288 \quad \text{or} \quad \frac{0.5}{u_*} = 2.288$$

$$u_* = \frac{0.5}{2.288}$$

$$= 0.2185 \text{ m/s}$$

$\rho_w = 1000$ kg/m³

using the relation $u_* = \sqrt{\tau_0 / \rho}$

$$0.2185 = \sqrt{\frac{\tau_0}{1000}}$$

$$0.2185^2 = \frac{\tau_0}{1000}$$

$$= 0.0477$$

$$\tau_0 = 0.0477 \times 1000$$

$$\boxed{\tau_0 = 47.676 \text{ N/m}^2}$$