DIMENSIONLESS NUMBERS

2-12

These Numbers are obtained by dividing the inertia force by viscous force or gravity force or Pressure force or surface tension force or elastic force.

As this is a ratio of one force to the other force, it will be dimensionless number. These Numbers also called as Dimensionless parameters

Important Dimensionless Numbers are

(1) Reynold's number (2) Froude's number
(3) Euler's number (4) Weber's number
(5) Mach's number

(i) Reynold's Number (Re)

It is defined as the ratio of inertia force of a flowing fluid and the viscous force of the fluid.

Inertia force (F_i) = Mass × Acceleration of flowing fluid

$= \rho \times \text{volume} \times \frac{\text{velocity}}{\text{Time}}$

$= \rho \times \frac{\text{volume}}{\text{Time}} \times \text{velocity}$

$= \rho \times A V \times V$

$= \rho A V^2$

$A \times \frac{m}{s}$

$A \times V$

$= \rho . V . \frac{V}{T}$

$= \rho . A . \frac{L}{T} . V$ $\quad \frac{L}{T} = V$ (m/s)

$= \rho . A . V . V = \rho A V^2$

Viscous force (F_v) = Shear Stress × Area

$$\left\{ \begin{array}{l} \tau = \mu \frac{du}{dy} \\ \text{Force} = \tau \times \text{Area} \end{array} \right\}$$

$$= \tau \times A$$

$$= \left(\mu \frac{du}{dy}\right) \times A$$

$$= \mu \cdot \frac{V}{L} \times A \left\{ \because \frac{du}{dy} = \frac{V}{L} \right\}$$

By definition Reynolds number

$$Re = \frac{F_i}{F_v}$$

$$= \frac{\rho A V^2}{\mu \cdot \frac{V}{L} \times A}$$

$$= \frac{\rho V L}{\mu}$$

$$= \frac{V L}{\left(\frac{\mu}{\rho}\right)}$$

$$Re = \frac{VL}{\nu} \quad \left\{ \frac{\mu}{\rho} = \nu \text{ Kinematic viscosity.} \right\}$$

In case of pipe flow, the linear dimension L is taken as diameter d Hence Reynolds Number for pipe flow

$$\boxed{Re = \frac{V \times d}{\nu} \quad (\text{or}) \quad \frac{\rho V d}{\mu}}$$

(2) Froude's Number (Fe) 2 - 14

The froude's number is defined as the square root of the ratio of inertia force of a flowing fluid to the gravity force

Mathematically

$$Fe = \sqrt{\frac{F_i}{F_g}} \rightarrow \text{from Re proof.}$$

F_g = Force due to gravity

= Mass × Acceleration due to gravity

= $\rho \times$ volume $\times g$

$= \rho \times l^3 \times g$

$= \rho \times l^2 \times l \times g$

$= \rho \times A \times L \times g$

$$Fe = \sqrt{\frac{F_i}{F_g}} = \sqrt{\frac{\rho A V^2}{\rho A L g}} = \sqrt{\frac{V^2}{Lg}} = \frac{V}{\sqrt{Lg}}$$

Euler's Number (Eu)

It is defined as the square root of the ratio of the inertia force of a flowing fluid to the pressure force.

$$Eu = \sqrt{\frac{F_i}{F_p}}$$

F_p = Intensity of Pressure × Area = $P \times A$

F_i = $\rho A V^2$

$$Eu = \sqrt{\frac{\rho A V^2}{P \times A}}$$

$$= \sqrt{\frac{V^2}{P/\rho}}$$

$$= \frac{V}{\sqrt{P/\rho}}$$

Weber's Number (We) It is defined as the square root of the ratio of the inertia force of a flowing fluid to the surface tension force

$$We = \sqrt{\frac{F_i}{F_s}}$$

F_i = Inertia force $\rho A V^2$

F_s = Surface Tension force

F_s = Surface tension force
= Surface tension per unit length × Length

$= \sigma \times L$

$$W_e = \sqrt{\frac{\rho A V^2}{\sigma \times L}}$$

$$= \sqrt{\frac{\rho \times L^2 \times V^2}{\sigma \times L}}$$

$$= \sqrt{\frac{\rho L \times V^2}{\sigma}}$$

$$= \sqrt{\frac{V^2}{\frac{\sigma}{\rho L}}}$$

$$W_e = \frac{V}{\sqrt{\frac{\sigma}{\rho L}}}$$

Mach's Number (M)

Square root of the ratio of the inertia force of a flowing fluid to the elastic force.

$$M = \sqrt{\frac{F_i}{F_e}}$$

$F_i = \rho A V^2$

$F_e =$ Elastic Stress $\times$ Area $= K \times A = K \times L^2$

$$M = \sqrt{\frac{\rho A V^2}{K L^2}}$$

$$= \sqrt{\frac{\rho L^2 V^2}{K L^2}}$$

$$= \sqrt{\frac{V^2}{\frac{K}{\rho}}}$$

$$= \frac{V}{\sqrt{\frac{K}{\rho}}}$$ → velocity of sound in the fluid

$$\boxed{M = \frac{V}{C}}$$

Mach Number Considered for the most of the Aero and Atmosphere air penetration through Jet and air problems.