2-18

MODEL LAWS or SIMILARITY LAWS

Models are designed on the basis of ratio of the force which is dominating in the phenomenon

The laws on which the models are designed for dynamic similarity are called model laws (or) Law of Similarity

1. Reynold's model Law

Law in which models are based on Reynold's number. it includes (i) pipe flow (ii) Resistance experienced by sub-marine, airplanes, fully immersed bodies.

As defined earlier that Reynold Number is the ratio of inertia force and viscous force, and hence fluid flow problem where viscous forces alone are predominant the models are designed for dynamic similarity on Reynolds Law, which says that the Reynolds number for the model must be equal to the Reynold number for the Prototype

V_m = Velocity of fluid in model
ρ_m = Density of fluid in model
L_m = Length of linear dimension of the model
μ_m = viscosity or fluid in model

and V_p ρ_p L_p and μ_p are the corresponding values of velocity, density, linear dimension and viscosity of fluid in prototype, then according to Reynold's model law

$$(R_e)_m = (R_e)_p$$

(or)
$$\frac{\rho_m V_m L_m}{\mu_m} = \frac{\rho_p V_p L_p}{\mu_p}$$

$$\text{(or)} \quad \frac{\rho_p V_p L_p}{\rho_m V_m L_m} \times \frac{1}{\frac{\mu_p}{\mu_m}} = 1$$

(or)

$$\frac{\rho_r V_r L_r}{\mu_r} = 1$$

where $\rho_r = \dfrac{\rho_p}{\rho_m}$ $V_r = \dfrac{V_p}{V_m}$

$L_r = \dfrac{L_p}{L_m}$, $\dfrac{\mu_p}{\mu_m} = \mu_r$

And also ρ_r V_r L_r and μ_r are called the scale ratios for density, velocity linear dimensions and viscosity

The scale ratios for time, acceleration, force and discharge for Reynold's model law are obtained as $\because V = \frac{m}{s} = \frac{L}{T}$; $\boxed{T = \frac{L}{V}}$

t_r = Time scale ratio $\frac{L_r}{V_r}$ $\quad \boxed{V = \frac{L}{T}}$

$$\left\{ \begin{array}{l} V = \frac{L}{t} \\ t = \frac{L}{V} \end{array} \right.$$

a_r = accelation scale ratio $\frac{V_r}{t_r}$

F_r = Force scale ratio

= Mass × Acceleration

$= m_r \times a_r$

$= \rho_r A_r V_r \times a_r$ $\qquad A_r$ = Area ratio

$= \rho_r L_r^2 V_r \times a_r$

Q_r = Discharge scale ratio $(\rho A V)_r$

$= \rho_r A_r V_r$

$= \rho_r L_r^2 V_r.$

Froude model Law

Froude model Law is the Law in which the models are based on froude number which means for dynamic similarity between the model and prototype, the Froude number for both of them should be equal.

Froude model law is applicable when the gravity force is only Predominant force which controls the flow in addition to the force of inertia

Froude model law is applied in the following fluid flow problems.

1. Free surface flows such as flow over spillways weirs, sluices, channels etc
2. Flow of jet from an orifice or nozzle
3. where waves are likely to be formed on surface
4. where fluids of different densities flow over one another

3-22

V_m = Velocity of fluid in model

L_m = Linear dimension or length of model

g_m = Acceleration due to gravity at a place where model is tested.

and V_p, L_p and g_p are the corresponding values of the velocity, length and acceleration due to gravity for the Prototype. Then according to Froude model Law

$$(F_e)_{model} = (F_e)_{Prototype}$$

(or)

$$\frac{V_m}{\sqrt{g_m L_m}} = \frac{V_p}{\sqrt{g_p L_p}}$$

If the tests on the model are performed on the same place i.e same height or altitude. where prototype is to operate, then $g_m = g_p$ and equation becomes

$$\frac{V_m}{\sqrt{L_m}} = \frac{V_p}{\sqrt{L_p}}$$

$$\frac{V_m}{V_p} \times \frac{1}{\sqrt{\frac{L_m}{L_p}}} = 1$$

$$\frac{V_p}{V_m} = \sqrt{\frac{L_p}{L_m}} = \sqrt{L_r}$$

where L_r = scale ratio for Length

$\frac{V_p}{V_m} = V_r$ = Scale ratio for velocity

$$\frac{V_p}{V_m} = V_r = \sqrt{L_r}$$

Scale ratio for various physical quantities based on Froude model Law are

(a) Scale ratio for time

$$\text{time} = \frac{\text{Length}}{\text{Velocity}}$$

Then ratio of time for prototype and model is

$$T_r = \frac{T_p}{T_m} = \frac{\left(\frac{L}{V}\right)_p}{\left(\frac{L}{V}\right)_m}$$

$$= \frac{\frac{L_p}{V_p}}{\frac{L_m}{V_m}}$$

$$= \frac{L_p}{L_m} \times \frac{V_m}{V_p}$$

$$= L_r \times \frac{1}{\sqrt{L_r}}$$

$$\boxed{T_r = \sqrt{L_r}}$$

$$= \frac{L_p}{V_p} \times \frac{V_m}{L_m}$$

$$= \frac{L_p}{L_m} \times \frac{V_m}{V_p}$$

$$= L_r \times \sqrt{V_r}$$

$$\because \frac{V_p}{V_m} = \sqrt{L_r}$$

$$1 - \frac{1}{2} = \frac{2-1}{2} = \frac{1}{2}$$

(b) Scale ratio for acceleration

Acceleration $\frac{V}{T}$

$$a_r = \frac{a_p}{a_m}$$

$$= \frac{\left(\frac{V}{T}\right)_p}{\left(\frac{V}{T}\right)_m}$$

$$= \frac{V_p}{T_p} \times \frac{T_m}{V_m}$$

$$= \frac{V_p}{V_m} \times \frac{T_m}{T_p}$$

$\left\{ \frac{V_p}{V_m} = \sqrt{L_r} \cdot \frac{T_p}{T_m} = \sqrt{L_r} \right.$

$$= \sqrt{L_r} \times \frac{1}{\sqrt{L_r}}$$

$$\boxed{a_r = 1.}$$

$\frac{1}{2} - \frac{1}{2} = 0 = 1.$

(c) scale ratio for discharge

$$Q = A \times V$$
$$= L^2 \times \frac{L}{T} = \frac{L^3}{T}$$

$$Q_r = \frac{Q_p}{Q_m}$$

$$= \frac{\left(\frac{L^3}{T}\right)_p}{\left(\frac{L^3}{T}\right)_m} = \left(\frac{L_p}{L_m}\right)^3 \times \left(\frac{T_m}{T_p}\right)$$

$$= L_r^3 \times \frac{1}{\sqrt{L_r}}$$

$3 - \frac{1}{2}$

$\frac{6-1}{2}$

$\frac{5}{2} = 2.5$

$$\boxed{Q_r = L_r^{2.5}}$$

(d) Scale ratio for force

Force = Mass × Acceleration

$$= \rho L^3 \times \frac{V}{T}$$

$$= \rho L^2 \cdot \frac{L}{T} \cdot V \qquad \frac{L}{T} = V$$

$$\boxed{\text{Force} = \rho L^2 V^2}$$

$= \frac{\text{mass}}{V} \times V$

$\rho \times L^3 \times \frac{V}{T}$

$= \rho \times L^3 \times \frac{V}{T}$

$= \rho L^2 V$

$= \rho V L^2 V$

Ratio for force,

$$F_r = \frac{F_p}{F_m}$$

$$= \frac{\rho_p L_p^2 V_p^2}{\rho_m L_m^2 V_m^2}$$

$$= \frac{\rho_p}{\rho_m} \left(\frac{L_p}{L_m}\right)^2 \times \left(\frac{V_p}{V_m}\right)^2$$

If the fluid used in model and prototype is same then..

$$\frac{\rho_p}{\rho_m} = 1 \quad (\text{or}) \quad \rho_p = \rho_m$$

and hence

$$F_r = \left(\frac{L_p}{L_m}\right)^2 \times \left(\frac{V_p}{V_m}\right)^2$$

$$= L_r^2 \times \left(\sqrt{L_r}\right)^2$$

$$= L_r^2 \cdot L_r$$

$$\boxed{F_r = L_r^3}$$

(e) Scale ratio for Pressure intensity

$$P = \frac{Force}{Area}$$

$$= \frac{\rho L^2 V^2}{L^2}$$

$$\boxed{P = \rho V^2}$$

Pressure ratio $P_r = \frac{P_p}{P_m} = \frac{\rho_p V_p^2}{\rho_m V_m^2}$

If fluid is same, then $\rho_p = \rho_m$

$$P_r = \frac{V_p^2}{V_m^2}$$

$$= \left(\frac{V_p}{V_m}\right)^2$$

$$P_r = \sqrt{L_r}^2 = \sqrt{L_r}^2 \quad \boxed{P_r = L_r}$$

(f) Scale ratio for work (or) energy (or) torque moment etc.

Torque = Force × Distance = F×L

Torque ratio $T_r^* = \frac{T_p^*}{T_m^*} = \frac{(F \times L)_p}{(F \times L)_m}$

$$= F_r \times L_r$$

$$= L_r^3 \times L_r$$

$$\boxed{\text{Torque } T_r^* = L_r^4}$$

(9) Scale ratio for Power

Power = Work per Unit time

$$= \frac{F \times L}{T}$$

$3 + \frac{1}{2} + \frac{1}{2} - \frac{1}{2}$

$\frac{6+1}{2} = \frac{7}{2} = 3.5$

Power ratio $P_r = \frac{P_p}{P_m}$

$$= \frac{\frac{F_p \times L_p}{T_p}}{\frac{F_m \times L_m}{T_m}}$$

$$P_r = \frac{F_p}{F_m} \times \frac{L_p}{L_m} \times \frac{1}{\frac{T_p}{T_m}}$$

$$P_r = F_r . L_r . \frac{1}{T_r}$$

$$= L_r^3 . L_r . \frac{1}{\sqrt{L_r}}$$

$$\boxed{P_r = L^{3.5}}$$

Power ratio

$3 + 1 - \frac{1}{2}$

$4 - \frac{1}{2}$

$6 - 1$

$= \frac{5}{2}$

$= \frac{5}{2}$

$= 2.5$

$3 + 1 - \frac{1}{2}$.

$3 + \frac{1}{2}$.

$\frac{6+1}{2} = \frac{7}{2}$.

$= 3.5$

(4)

Problem on Froude model Law 2-28

In 1 in 40 model of a Spillway, the velocity and discharge are 2 m/s and 2.5 m^3/s. Find the Corresponding velocity and discharge in the Prototype

Given

Scale ratio of Length $L_r = 40$

Velocity in model $V_m = 2 m/s$

Discharge in model $Q_m = 2.4\ m^3/s$

Let V_p and Q_p are the velocity and discharge in Prototype

Using equation for velocity ratio $\frac{V_p}{V_m} = \sqrt{L_r}$

$= \sqrt{40}$

$$V_p = V_m \times \sqrt{40}$$

$$= 2 \times \sqrt{40}$$

$$= 12.65\ m/s$$

Using equation for discharge ratio

$$\frac{Q_p}{Q_m} = L_r^{2.5} = (40)^{2.5}$$

$$Q_p = Q_m \times 40^{2.5}$$

$$= 2.5 \times 40^{2.5}$$

$$= 25298.2\ m^3/s$$