Velocity triangles

It is the component which is responsible for "actual work done" across blades.

The jet of fluid that strikes the turbine blades, has its two components:

1. Whirl component
2. Axial thrust

The whirl velocity is the tangential component of absolute velocity at the blade inlet and outlet. This component of velocity is responsible for the whirling or rotating of the turbine rotor.

Otherwise

In turbomachinery, a velocity triangle or a velocity diagram is a triangle representing the various components of velocities of the working fluid in a turbomachine.

Velocity triangles may be drawn for both the inlet and outlet sections of any turbomachine.

The vector nature of velocity is utilized in the triangles, and the most basic form of a velocity triangle consists of the tangential velocity, the absolute velocity and the relative velocity of the fluid making up three sides of the triangle.

A general velocity triangle consists of the following vectors:

V : Absolute Velocity of The Fluid.

U : Blade Linear Velocity.

V_r: Relative Velocity of The Fluid After Contact With Rotor.

V_w: Tangential Component of V (Absolute Velocity), Called Whirl Velocity.

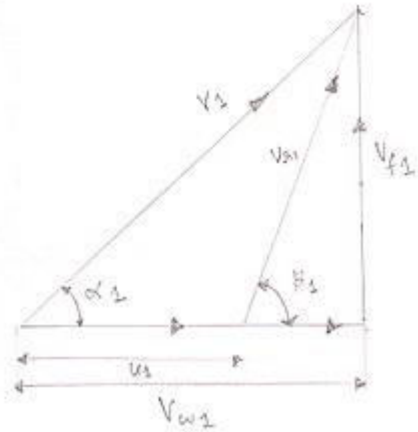
V_f: Flow Velocity

(Axial Component In Case of Axial Machines, Radial Component In Case of Radial Machines).

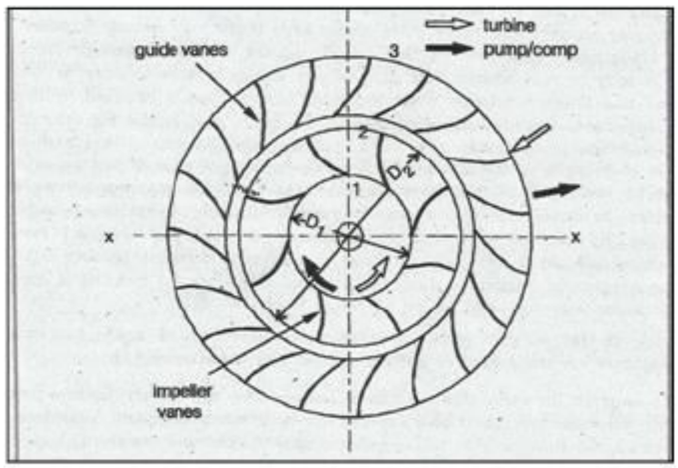
The Following Angles Are Encountered During The Analysis:

A: Angle Made by V With The Plane of The Machine (Usually The Nozzle Angle or The Guide Blade Angle).

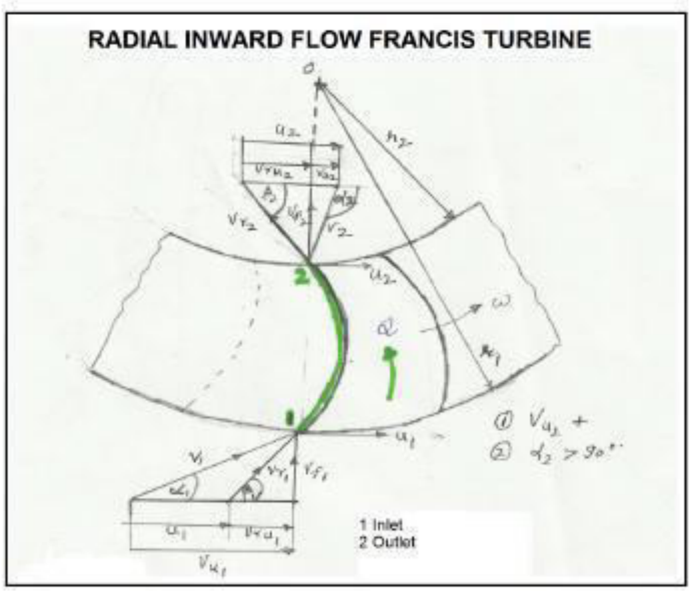
B: Angle of The Rotor Blade. Absolute Angle



V1
Vf1
u1
Vw1



turbine
pump/comp
guide vanes
3
2
1
D2
D1
x
x
impeller
vanes



RADIAL INWARD FLOW FRANCIS TURBINE
2
1
ω
1 Inlet
2 Outlet

Vu2 = Whirl component of absolute velocity at outlet;

Vf2 = Flow component of absolute velocity at outlet ; Inlet and Outlet Velocity Triangles

Referring to velocity triangles

1 – Inlet, 2 – outlet

V1 = Absolute velocity of the fluid at inlet (before entering the rotor vanes)

Vr1 = Relative velocity of the fluid at rotor inlet

Vu1 = Tangential component of absolute velocity

OR

Whirl component of velocity at inlet

Vru2 = Whirl component of relative velocity at outlet

U2 = Linear rotor velocity at outlet

1 = Fluid or jet angle at outlet (To the direction of wheel rotation)

1 = Vane (blade) angle at outlet (To the direction of wheel rotation)

Vf1 = Flow component of absolute velocity at inlet

Vru1 = Whirl component of relative velocity at inlet

U1 = Linear rotor vane velocity at inlet

1= Absolute jet angle at inlet

1 = Vane (blade) angle at inlet

Referring to outlet velocity triangle

The above figure shows the velocity triangles at entry and exit of a general TM. The angular velocity of the rotor is ώ rad/sec and is given by

$$\omega = \frac{2\pi N}{60} - - - 1$$

The peripheral velocity of the blade at the entry and exit corresponding to the diameter D_1 *and* D_2 are given by

$$u_1 = \frac{\pi D_1 N}{60} - - - 2$$

$$u_2 = \frac{\pi D_2 N}{60} - - - 3$$

The three velocity vector V, u and V_r at the section are related by a simple vector equation

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The three velocity vector V, u and V_r at the section are related by a simple vector equation

$$V = u + V_r --- 4$$

Considering unit mass of the fluid entering and leaving in unit time we have

Angular momentum of the inlet $= V_{u1} \times R_1 ----5$

Similarly

Angular momentum of the outlet $= V_{u2} \times R_2 ----6$

Torque produced = rate of change of Angular momentum

$$T = \{\text{Angular momentum at inlet}\} - \{\text{Angular momentum at outlet}\}$$

$$T = (V_{u1}R_1) - (V_{u2}R_2) --- 7$$

Therefore the work done is given by

$$W = Torque \times angular\ velocity\ of\ the\ rotor$$

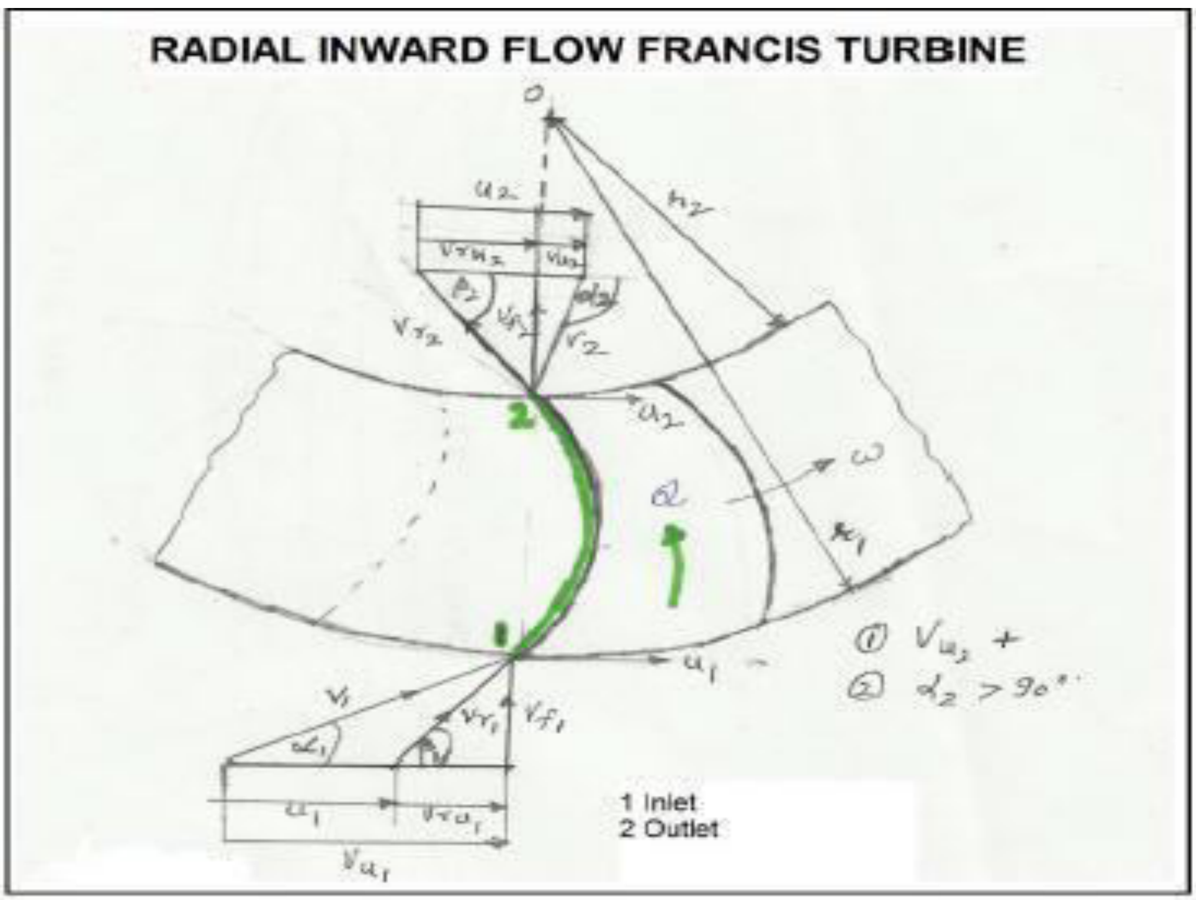
$$W = (V_{u1}R_1 - V_{u2}R_2)\omega$$

$$= [V_{u1}(\omega R_1) - V_{u2}(\omega R_2)]$$

$$= \left[V_{u1}\left(\frac{2\pi N}{60}R_1\right) - V_{u2}\left(\frac{2\pi N}{60}R_2\right)\right]$$

$$W = \left[\frac{V_{u1}\pi D_1 N}{60} - \frac{V_{u2}\pi D_2 N}{60}\right]$$

$$W = [V_{u1}u_1 - V_{u2}u_2] - - - 8$$



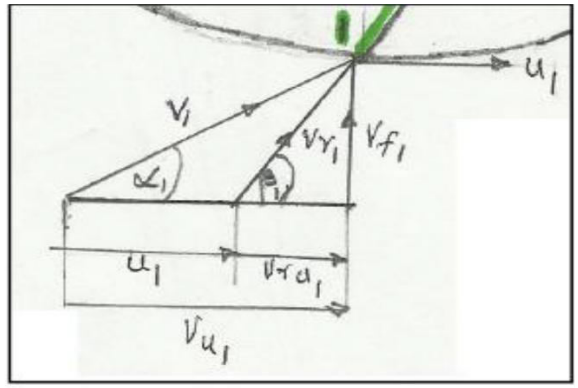
From inlet velocity triangle

$V_{f1}^2 = V_1^2 - V_{u1}^2$

$V_{r1}^2 = V_{f1}^2 + V_{ru1}^2$

$V_{r1}^2 = V_1^2 - V_{u1}^2 + (V_{u1} - U_1)^2$

$= V_1^2 - \cancel{V_{u1}^2} + \cancel{V_{u1}^2} - 2V_{u1}U_1 + U_1^2$



Rearranging

$2V_{u1}U_1 = V_1^2 + U_1^2 - V_{r1}^2$

$$\boxed{V_{u1}U_1 = \frac{V_1^2 + U_1^2 - V_{r1}^2}{2}} \quad m^2/s^2 \text{ OR } Nm/kg... (1)$$

From outlet velocity triangle

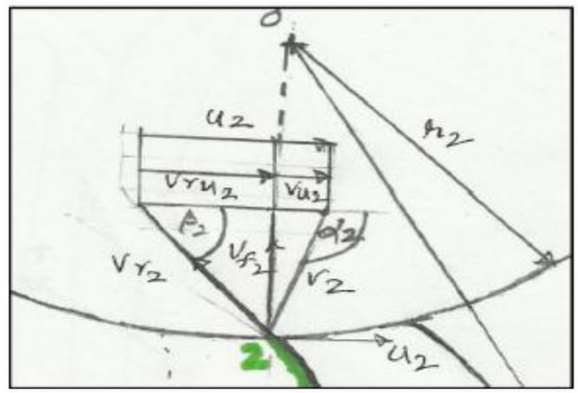
$V_{r2}^2 = V_{ru2}^2 + V_{f2}^2$

$= (U_2 - V_{u2})^2 + (V_2^2 - V_{u2}^2)$

Taking $V_{ru2} = (U_2 - V_{u2})$ in magnitude only and not in directions

$V_{r2}^2 = U_2^2 - 2V_{u2}U_2 + \cancel{V_{u2}^2} + V_2^2 - \cancel{V_{u2}^2}$

$$\therefore \boxed{V_{u2}U_2 = \frac{V_2^2 + U_2^2 - V_{r2}^2}{2}} \quad m^2/s^2 \text{ OR } Nm/kg... (2)$$



CASE 1:

Taking direction of rotation as positive

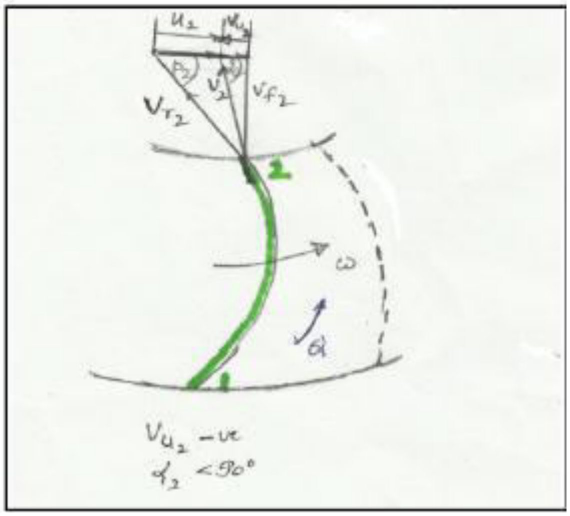
V_{u1} +ve and V_{u2} also +ve.

Work done/kg or Energy transfer in Turbine

Work done/kg = $(V_{u1}U_1 - V_{u2}U_2)$

$$\text{Energy Transfer (E)} = \left[\frac{V_1^2 + U_1^2 - V_{r1}^2}{2}\right] - \left[\frac{V_2^2 + U_2^2 - V_{r2}^2}{2}\right]$$

$$= \left[\frac{V_1^2 - V_2^2}{2}\right] + \left[\frac{U_1^2 - U_2^2}{2}\right] + \left[\frac{V_{r2}^2 - V_{r1}^2}{2}\right]$$



Components of energy transfer

1. $\frac{V_1^2 - V_2^2}{2}$ is change in absolute kinetic energy in m^2/s^2 or Nm/kg
2. $\frac{U_1^2 - U_2^2}{2}$ is change in centrifugal energy of fluid felt as static pressure change in rotor blades in m^2/s^2 or Nm/kg
3. $\frac{V_{r2}^2 - V_{r1}^2}{2}$ Is change in relative velocity energy felt as static pressure change in rotor blades in m2/s2 or Nm/kg

	Impulse	Reaction
1.	All potential energies are converted into KE by nozzle before entering to turbine runner	only a portion of the fluid energy is transferred into KE before the fluid enters the turbine.
2.	Flow regulation is possible without losses	Flow regulation is possible with loss
3.	Flow is regulated by means of a needle valve fitted into the nozzle	Flow is regulated by means of a guide-vane assembly
4.	water may be allowed to enter a part or whole of the wheel circumference	water is admitted over the circumference of the wheel
5.	wheel does not run full and air has free access to the buckets	water completely fills the vane passages throughout the operation of turbine.
6	Unit is installed above the tailrace	Unit is kept entirely submerged in water below tailrace.
7.	Blades are only in action when they are in front of nozzle	Blades are in action at all the time.

Tangential Flow Turbine:

water flows along the tangent to the path of the runner.

Ex: Pelton wheel.

Radial flow turbine:

In a radial flow turbine, water flows along the radial direction and mainly in the plane normal to the axis of rotation, as it passes through the runner.

It may be either inward radial flow type or outward radial flow type.

Inward radial flow:

water enters at the outer circumference and flows radially inwards to the centre of the runner

Ex: old francis turbine, Thomson turbine.

outward radial flow:

water enters at the centre and flows radially outwards towards the outer periphery of the runner

Ex: Fourneyron turbine.

Mixed Flow turbine

In a mixed flow turbine, the water enters the blades radially and comes out axially and parallel to the turbine shaft

Ex: Modern Francis turbine

In our subject point of view, the following turbines are important and will be discussed one by one.

1. pelton wheel 2. Francis turbine 3. Kaplan turbine

Axial Flow Turbine:

water flows parallel to the axis of the turbine shaft

Eg. 1) Kaplan Turbine, 2) propeller Turbine.

V – velocity of Jet (absolute)

V_w – wirl velocity

V_r – Relative velocity (when there is no friction)

u – Vane velocity (Runner)

φ – Angle of blade at outlet (or) Vane angle outlet

β – Angle made by V_{r2} with the direction of motion of the vane at outlet.

V_f – velocity of flow

d – dia of Jet

D – dia of wheel

D* – dia of penstock

Question and answers: -

Draw velocity triangles at inlet and outlet of typical Francis turbine vane.

Ans. There are three types of velocity triangles for. inlet and outlet in Francis turbine. Triangles are made for slow runner, medium runner and fast runner.

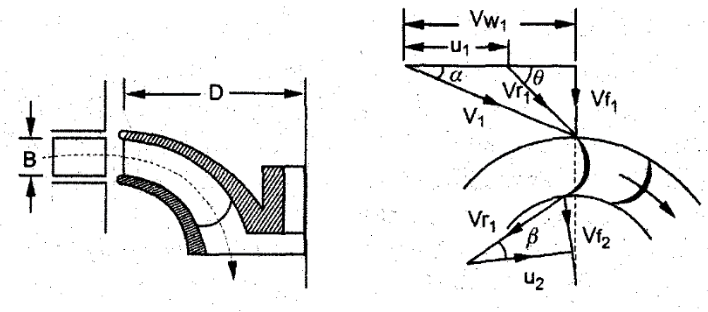


Fig. Slow runner

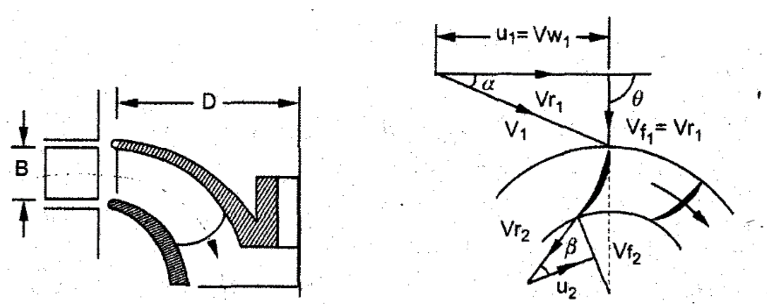


Fig. Medium Runner

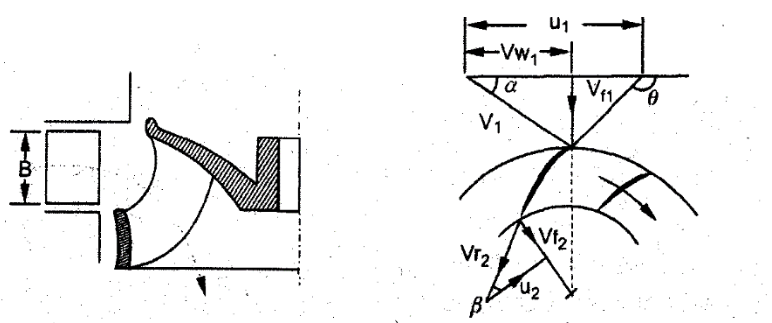


Fig. Fast Runner