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DEPARTMENT OF MATHEMATICS

3) Find the Fourier transform of $-f(x) = \begin{cases} a^2 - x^2, \ |x| < a \\ 0, \ |x| > a > 0. \end{cases}$ Hence deduce that (i) $\int \frac{Sint - t \cos t}{t^3} dt = \frac{\pi}{4}$ $(ii) \int_{-\frac{1}{2}}^{\infty} \left(\frac{sint - t cost}{t^3}\right)^2 dt = \frac{\pi}{15}$ Solution : The Fourier transform of f(n) is, $F[f(x)] = F(s) = \frac{1}{\sqrt{2\pi}} \int -f(x) e^{isx} dx$ $= \int_{2\pi} \int (a^2 - x^2) e^{isx} dx$ $= \frac{1}{\sqrt{2\pi}} \int (a^2 - x^2) (\cos sx + i \sin sx) dx$ $= \frac{1}{\sqrt{2\pi}} \int_{-a}^{a} (a^2 - x^2) \cos s x \, dx + \int_{a}^{a} \int_{a}^{a} (a^2 - x^2) \sin s x \, dx$ $= \frac{1}{\sqrt{a^{2}}} \cdot 2 \int (a^{2} - x^{2}) \cos sx \, dx$





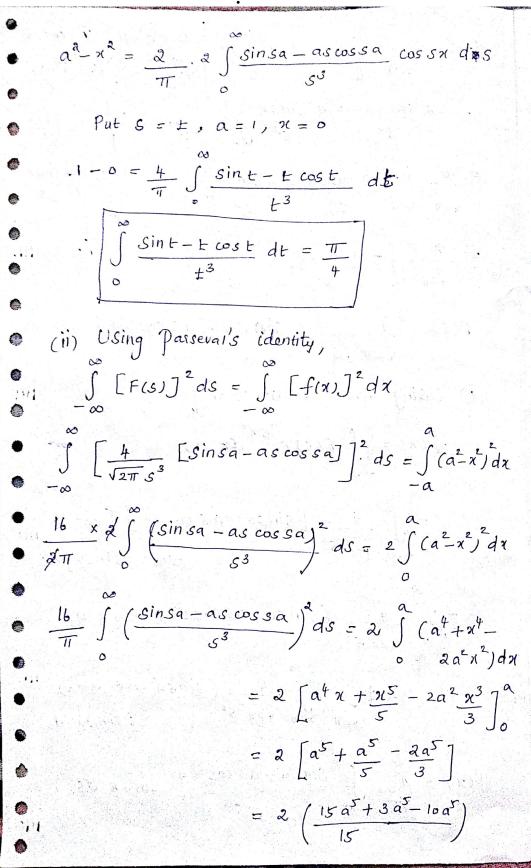
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$$F(s) = \frac{2}{\sqrt{2\pi}} \left\{ (a^{*} - x^{*}) \frac{\sin sx}{s} \\ -2x \frac{\cos sx}{s^{*}} + 2 \frac{\sin sx}{s^{*}} \int_{a}^{a} \\ \frac{u^{*}}{z} = -2x \\ \frac{u^{*}}{u^{*}} = -2x \\ \frac{u^{*}}{s^{*}} + 2 \frac{\sin sx}{s^{*}} \int_{a}^{a} \\ \frac{u^{*}}{u^{*}} = -2x \\ \frac{u^$$



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