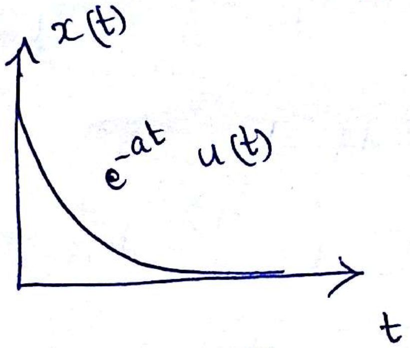
PROBLEMS :-

1) Find the Fourier transform of decaying Exponential function



$$x(F) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi ft}\, dt$$

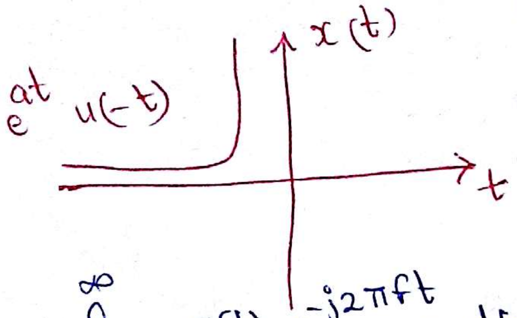
$$= \int_{-\infty}^{\infty} e^{-at}\, u(t)\, e^{-j2\pi ft}\, dt$$

$$= \int_{0}^{\infty} e^{-at}\, e^{-j2\pi ft}\, dt$$

$$= \int_{0}^{\infty} e^{-(a+j2\pi f)t}\, dt \Rightarrow \left[\frac{e^{-(a+j2\pi f)}}{-(a+j2\pi f)}\right]_{0}^{\infty}$$

$$x(F) = \frac{1}{a+j2\pi f}$$

2) Find the Fourier transform of rising Exponential f



$$X(F) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi ft}\, dt$$

$$= \int_{-\infty}^{0} e^{+at}\, e^{-j2\pi ft}\, dt$$

$$= \int_{-\infty}^{0} e^{t(a-j2\pi f)}\, dt \Rightarrow \left[\frac{e^{(a-j2\pi f)t}}{a-j2\pi f}\right]_{-\infty}^{0}$$

$$X(F) = \frac{1}{a-j2\pi f}$$

HW

3) Find the Fourier transform of $x(t) = e^{-0.5t}\, u(t)$

$$X(F) = \frac{1}{0.5+j2\pi f}$$

4) Find the Fourier transform of $u(t)$

$$X(F) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi ft}\, dt$$

$$= \int_{-\infty}^{\infty} u(t)\, e^{-j2\pi ft}\, dt$$

$$= \int_{0}^{\infty} e^{-j2\pi ft}\, dt$$

$$= \left[\frac{e^{-j2\pi f}}{-j2\pi f}\right]_{0}^{\infty}$$

$$X(F) = \frac{1}{j2\pi f}$$

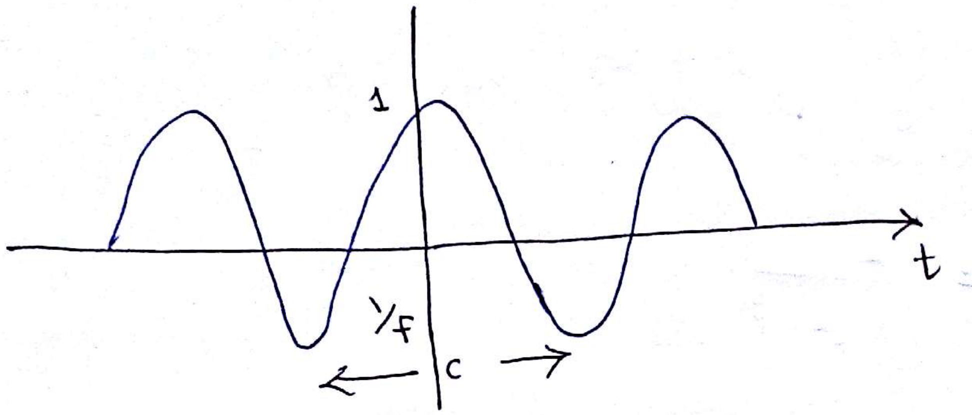
Signum Function :-

$$\text{Signum}(t) = \begin{cases} 1, & t > 0 \\ -1, & t < 0 \end{cases}$$

Sinc Function :-

$$\text{sinc}\, x = \frac{\sin \pi x}{\pi x}$$

5) Find the fourier transform of cosine wave as shown in fig



$$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}.$$

$$x(t) = \cos 2\pi f_c t$$

$$X(F) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi ft}\, dt$$

$$= \int_{-\infty}^{\infty} \cos 2\pi f_c t\; e^{-j2\pi ft}\, dt$$

$$= \int_{-\infty}^{\infty} \frac{e^{j2\pi f_c t} + e^{-j2\pi f_c t}}{2}\; e^{-j2\pi ft}\, dt$$

$$= \frac{1}{2} \int_{-\infty}^{\infty} \left[e^{-j2\pi t(f-f_c)} + e^{-j2\pi t(f+f_c)} \right] dt$$

$$= \frac{1}{2} \left[\int_{-\infty}^{\infty} e^{-j2\pi t(f-f_c)}\, dt + \int_{-\infty}^{\infty} e^{-j2\pi t(f+f_c)}\, dt \right]$$

Apply the result :- $\int_{-\infty}^{\infty} e^{-j2\pi(f-f_0)t} = \delta(f-f_0)$

$$X(F) = \frac{1}{2} \left[\delta(f-f_c) + \delta(f+f_c) \right]$$

6) Find the Fourier transform of sine wave :-

HW

$$x(t) = \sin 2\pi f_c t$$

$$\sin 2\pi f_c t = \frac{e^{j2\pi f_c t} - e^{-j2\pi f_c t}}{2j}$$

$$X(F) = \frac{1}{2j}\left[\delta(f-f_c) - \delta(f+f_c)\right]$$

7) Find the Fourier transform of $x(t) = A\cos(2\pi f_c t)\, u(t)$

$$X(F) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi f t}\, dt$$

$$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

$$= \int_{-\infty}^{\infty} A\cos(2\pi f_c t)\, u(t)\, e^{-j2\pi f t}\, dt$$

$$= A\int_{0}^{\infty} \frac{e^{j2\pi f_c t} + e^{-j2\pi f_c t}}{2}\, e^{-j2\pi f t}\, dt$$

$$= A/2 \int_{0}^{\infty} \left(e^{j2\pi f_c t} + e^{-j2\pi f_c t}\right) e^{-j2\pi f t}\, dt$$

$$= A/2 \left[\int_{0}^{\infty} e^{-j2\pi(f-f_c)t}\, dt + \int_{0}^{\infty} e^{-j2\pi(f+f_c)t}\, dt\right]$$

$$= A/2 \left[\frac{e^{-j2\pi(f-f_c)t}}{-j2\pi(f-f_c)} + \frac{e^{-j2\pi(f+f_c)t}}{j2\pi(f+f_c)}\right]_0^{\infty}$$

$$= A/2 \left[\frac{1}{j2\pi(f-f_c)} + \frac{1}{j2\pi(f+f_c)}\right]$$

$$= A/2 \cdot \frac{1}{j2\pi}\left[\frac{1}{f-f_c} + \frac{1}{f+f_c}\right]$$

$$= \frac{A}{j4\pi}\left[\frac{f + \cancel{f_c} + f - \cancel{f_c}}{f^2 - f_c^2}\right]$$

$$= \frac{A}{j\cancel{4}_2\pi}\left[\frac{\cancel{2}f}{f^2 - f_c^2}\right] = \frac{A}{j2\pi}\left[\frac{f}{f^2 - f_c^2}\right]$$

) Find the Fourier transform of $x(t) = A\sin(2\pi f_c t)$

H.W

$$X(F) = \frac{-A}{2\pi}\left[\frac{f_c}{f^2 - f_c^2}\right]$$

(9) Find the Fourier transform of $\cos^2(2\pi f_0 t)$.

$$X(F) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi ft}\, dt$$

$$= \int_{-\infty}^{\infty} \cos^2(2\pi f_0 t)\, e^{-j2\pi ft}\, dt$$

$\cos^2\theta = \frac{1+\cos 2\theta}{2}$

$$= \int_{-\infty}^{\infty} \left[\frac{1+\cos 4\pi f_0 t}{2}\right] e^{-j2\pi ft}\, dt$$

$$= \frac{1}{2}\left[\int_{-\infty}^{\infty} e^{-j2\pi ft}\, dt + \int_{-\infty}^{\infty} \cos 4\pi f_0 t\; e^{-j2\pi ft}\, dt\right]$$

$$= \frac{1}{2}\left[\delta(F) + \int_{-\infty}^{\infty} \cos 4\pi f_0 t\; e^{-j2\pi ft}\, dt\right]$$

$$= \frac{1}{2}\left[\delta(F) + \int_{-\infty}^{\infty} \frac{e^{j4\pi f_0 t} + e^{-j4\pi f_0 t}}{2}\, e^{-j2\pi ft}\, dt\right]$$

$$= \frac{1}{2}\left[\delta(F) + \frac{1}{2}\left[\int_{-\infty}^{\infty} e^{j4\pi f_0 t} + e^{-j4\pi f_0 t}\; e^{-j2\pi ft}\, dt\right]\right]$$

$$= \frac{1}{2}\left[\delta(F) + \frac{1}{2}\left[\int_{-\infty}^{\infty} e^{-j2\pi(f-2f_0)t} + e^{-j2\pi(f+2f_0)t}\right]\right]$$

$$= \frac{1}{2}\left[\delta(F) + \frac{1}{2}\left[\delta(F-2f_0) + \delta(f+2f_0)\right]\right]$$

$$X(F) = \frac{1}{2}\delta(F) + \frac{1}{4}\left[\delta(f-2f_0) + \delta(f+2f_0)\right]$$