1) Find the Inverse Laplace Transform of $X(S) = \frac{S^2+2S-2}{S(S+2)(S-3)}$ using partial fraction Method.

$$\frac{S^2+2S-2}{S(S+2)(S-3)} = \frac{A}{S} + \frac{B}{S+2} + \frac{C}{S-3}$$

$$S^2+2S-2 = A(S+2)(S-3) + B(S)(S-3) + C(S)(S+2)$$

when $S=0$

$-2 = -6A$

$\boxed{A = 1/3}$

when $S=-2$

$(-2)^2 + 2(-2) - 2 = B(-5)(-2)$

$4-4-2 = 10B$

$\boxed{B = -1/5}$

when $S=3$

$9+6-2 = 15C$

$13 = 15C$

$\boxed{C = 13/15}$

$$\frac{S^2+2S-2}{S(S+2)(S-3)} = \frac{1/3}{S} - \frac{1/5}{S+2} + \frac{13/15}{S-3}$$

$$= \frac{1}{3} L^{-1}\left(\frac{1}{S}\right) - \frac{1}{5} L^{-1}\left(\frac{1}{S+2}\right) + \frac{13}{15} L^{-1}\left(\frac{1}{S-3}\right)$$

$$x(t) = \frac{1}{3} u(t) - \frac{1}{5} e^{-2t} u(t) + \frac{13}{15} e^{3t} u(t)$$

2) Find the ILT of $X(S) = \frac{1}{S^2+3S+2}$ using Partial fraction Method :- ROC : $-2 < Re(S) < -1$

$$\frac{1}{S^2+3S+2} = \frac{A}{S+2} + \frac{B}{S+1}$$

$$1 = A(S+1) + B(S+2)$$

when $S=-2$

$\boxed{A = -1}$

when $S=-1$

$\boxed{B = 1}$

$$\therefore \frac{1}{(S+2)(S+1)} = \frac{-1}{S+2} + \frac{1}{S+1}$$

$$= (-1)\ L^{-1}\left(\frac{1}{S+2}\right) + (1)\ L^{-1}\left(\frac{1}{S+1}\right)$$

$$x(t) = -e^{-2t}\ u(t) + e^{-t}\ u(t)$$

Roc : $-2 < Re(S) < -1$

$Re(S) > -2$, $Re(S) < -1$

$$x(t) = \left[e^{-2t}\ u(t)\right] + \left[-e^{-t}\ u(t)\right]$$

③ $X(S) = \dfrac{4}{(S+2)(S+4)}$ Roc : (i) $-2 > Re(S) > -4$

(ii) $Re(S) < -4$

$$\frac{4}{(S+2)(S+4)} = \frac{A}{S+2} + \frac{B}{S+4}$$

$$4 = A(S+4) + B(S+2)$$

when $S = -4$

$4 = -2B$

$\boxed{B = -2}$

when $S = -2$

$4 = 2A$

$\boxed{A = 2}$

$$X(S) = \frac{2}{S+2} - \frac{2}{S+4}$$

$$= 2\ L^{-1}\left(\frac{1}{S+2}\right) - 2\ L^{-1}\left(\frac{1}{S+4}\right)$$

$$x(t) = 2\,e^{-2t}\ u(t) - 2\,e^{-4t}\ u(t)$$

ROC: $-2 > Re(S) > -4$

$Re(S) > -4$, $Re(S) < -2$

$$x(t) = \left[2e^{-4t}\,u(t)\right] + \left[2e^{-2t}\,u(-t)\right]$$

(4) HW $\quad X(S) = \dfrac{3S+7}{S^2-2S-3}$ (i) $Re(S) > 3$ (ii) $Re(S) < -1$

(iii) $-1 < Re(S) < 3$

(5) $X(S) = \dfrac{-3}{(S+2)(S-1)}$

$$\frac{-3}{(S+2)(S-1)} = \frac{A}{S+2} + \frac{B}{S-1}$$

$$-3 = A(S-1) + B(S+2)$$

when $S = 1$

$-3 = 3B$

$\boxed{B = 1}$

when $S = -2$

$-3 = -3A$

$\boxed{A = 1}$

$$X(S) = \frac{1}{S+2} + \frac{1}{S+1}$$

$$= L^{-1}\left(\frac{1}{S+2}\right) + L^{-1}\left(\frac{1}{S+1}\right)$$

$$x(t) = e^{-2t}\,u(t) + e^{-t}\,u(t)$$

(i) $-2 < Re(S) < 1$

$Re(S) > -2$ $\quad Re(S) < 1$

$$x(t) = e^{-2t}\,u(t) - e^{-t}\,u(-t)$$

(ii) $Re(S) > 1$

$$x(t) = e^{-t}\,u(t)$$

(iii) $Re(S) < -2$

$$x(t) = -1\,e^{-2t}\,u(-t)$$