SNS COLLEGE OF TECHNOLOGY

AN AUTONOMOUS INSTITUTION

COIMBATORE – 35

Department of

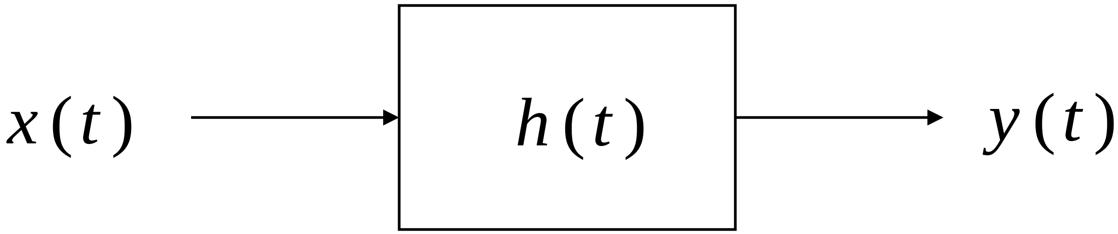
Electronics and Communication Engineering

UNIT III

FREQUENCY RESPONSE OF LTI CONTINUOUS TIME SYSTEMS

CT LTI Systems

- Consider the following CT LTI system:



- Assumption: the impulse response *h*(*t*) is ***absolutely integrable,*** i.e.,

$$\int | h(t) | dt < \infty$$

Frequency Response Analysis

- By the term *frequency response*, we mean the steady-state response of a system to a sinusoidal input.

- In frequency-response methods, we vary the frequency of the input signal over a certain range and study the resulting response.

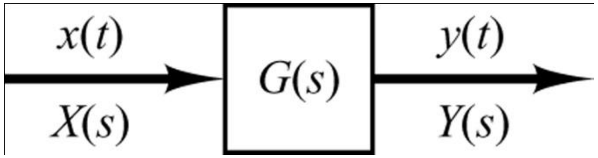
Advantages of the frequency-response approach

1. **We can use the data obtained from measurements on the physical system without deriving its mathematical model.**
2. **frequency-response tests are, in general, simple and can be made accurately by use of readily available sinusoidal signal generators and precise measurement equipment.**

Frequency Response Analysis

Obtaining Steady-State Outputs to Sinusoidal Inputs

☞ **The steady-state output of a transfer function system can be obtained directly from the sinusoidal transfer function, that is, the transfer function in which *s* is replaced by $j\omega$, where ω is frequency.**



- **If the input *x(t)* is a sinusoidal signal, the steady-state output will also be a sinusoidal signal of the same frequency, but with possibly different magnitude and phase angle.**

Response of a CT LTI System to a Sinusoidal Input

- What's the response $y(t)$ of this system to the input signal

$$x(t) = A\cos(\omega_0 t + \theta), \;\; t \in \mathbb{R} \qquad ?$$

- We start by looking for the response $y_c(t)$ of the same system to

$$x_c(t) = Ae^{j(\omega_0 t + \theta)} \qquad t \in \mathbb{R}$$

Response of a CT LTI System to a Complex Exponential Input

- The output is obtained through convolution as

$$y_c(t) = h(t) * x_c(t) = \int_{\square} h(\tau) x_c(t-\tau) d\tau =$$

$$= \int_{\square} h(\tau) A e^{j(\omega_0 (t-\tau)+\theta)} d\tau =$$

$$= \underbrace{A e^{j(\omega_0 t+\theta)}}_{x_c(t)} \int_{\square} h(\tau) e^{-j\omega_0 \tau} d\tau =$$

$$= x_c(t) \int_{\square} h(\tau) e^{-j\omega_0 \tau} d\tau$$

The Frequency Response of a CT LTI System

- By defining

$$H(\omega) = \int_{\square} h(\tau) e^{-j\omega\tau} d\tau$$

$H(\omega)$ **is the frequency response of the CT, LTI system = Fourier transform of *h*(*t*)**

it is

$$y_c(t) = H(\omega_0) x_c(t) =$$

$$= H(\omega_0) A e^{j(\omega_0 t + \theta)}, \quad t \in \square$$

- Therefore, the response of the LTI system to a complex exponential is another complex exponential with the same frequency ω_0

Analyzing the Output Signal $y_c(t)$

- Since is in general a complex quantity, we can write

$$y_c(t) = H(\omega_0) A e^{j(\omega_0 t + \theta)} =$$

$$= |H(\omega_0)| e^{j \arg H(\omega_0)} A e^{j(\omega_0 t + \theta)} =$$

$$= \underbrace{A|H(\omega_0)|}e^{j(\omega_0 t + \theta + \arg H(\omega_0))}$$

output signal's magnitude

output signal's phase

Frequency Analysis of an RC Circuit

1. The complex impedance of the capacitor is equal to $1/sC$ where
2. If the input voltage is $s = \sigma + j\omega$, then the output signal is given by $x_c(t) = e^{st}$

$$y_c(t) = \frac{1/sC}{R + 1/sC} e^{st} = \frac{1/RC}{s + 1/RC} e^{st}$$

Frequency Analysis of an RC Circuit

- Setting $s = j\omega_0$, it is

$$x_c(t) = e^{j\omega_0 t} \qquad y_c(t) = \frac{1/RC}{j\omega_0 + 1/RC} e^{j\omega_0 t}$$

whence we can write

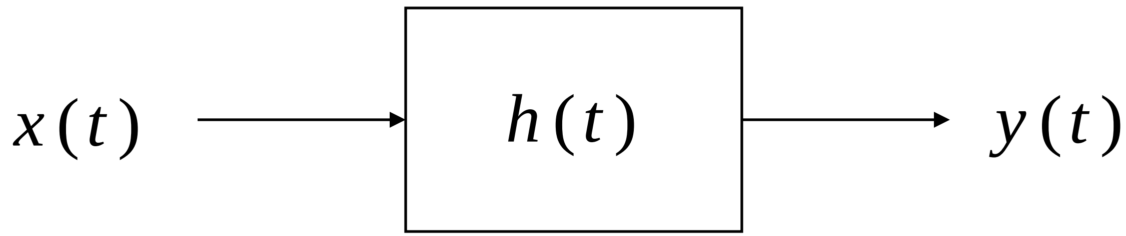
$$y_c(t) = H(\omega_0) x_c(t)$$

where

$$H(\omega) = \frac{1/RC}{j\omega + 1/RC}$$

Response of a CT LTI System to Aperiodic Inputs

- Consider the following CT, LTI system



- Its I/O relation is given by

$$y(t) = h(t) * x(t)$$

which, in the frequency domain, becomes

$$Y(\omega) = H(\omega) X(\omega)$$

Response of a CT LTI System to Aperiodic Inputs

- From $Y(\omega) = H(\omega)X(\omega)$, the magnitude spectrum of the output signal *y*(*t*) is given by

$$|Y(\omega)| = |H(\omega)||X(\omega)|$$

and its phase spectrum is given by

$$\arg Y(\omega) = \arg H(\omega) + \arg X(\omega)$$

Response of an RC Circuit to a Rectangular Pulse

- Consider the RC circuit

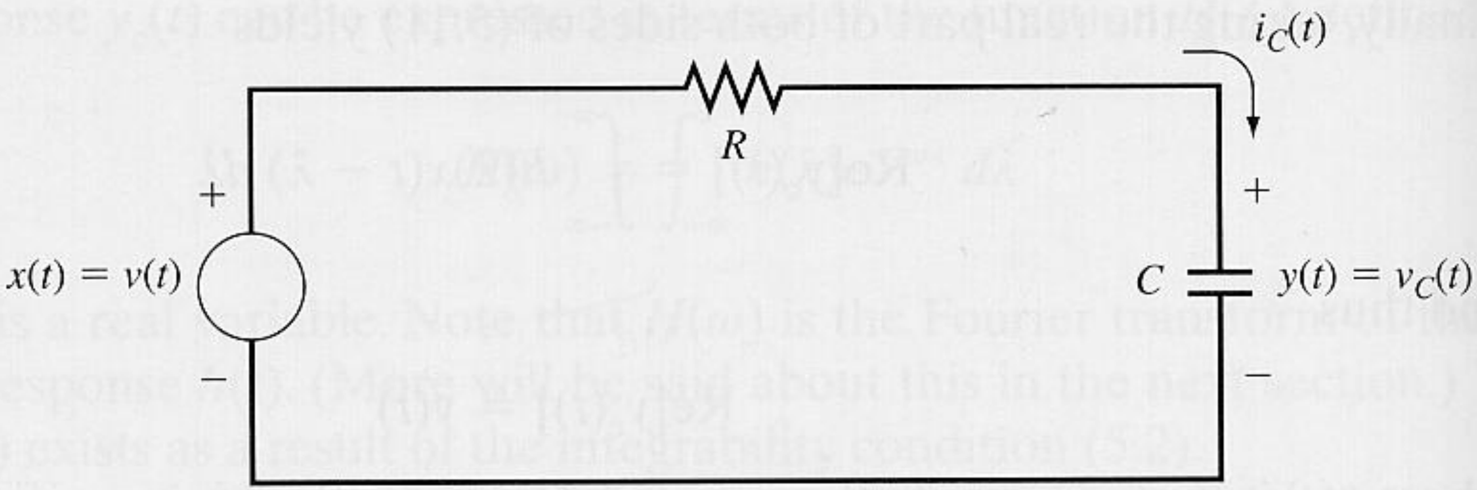


Figure 5.1 *RC* circuit in Example 5.2.

with input $x(t) = \text{rect}(t)$

Response of an RC Circuit to a Rectangular Pulse

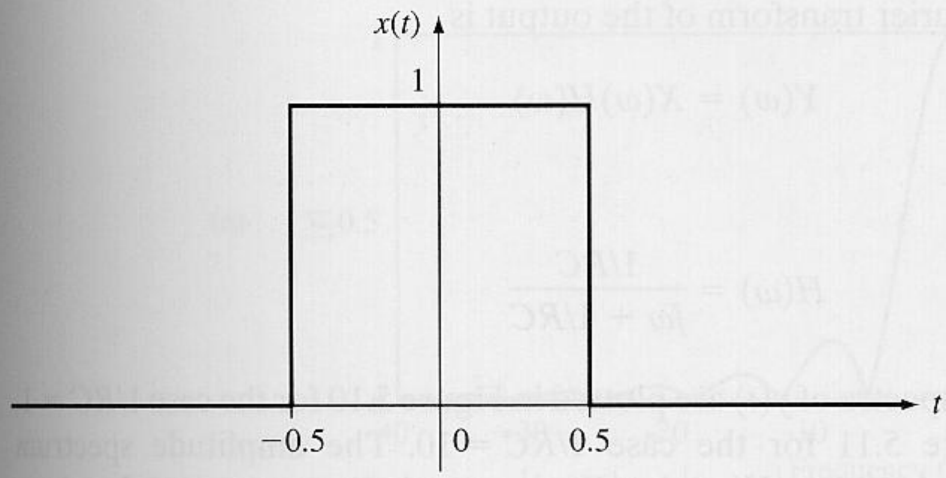


Figure 5.8 Input pulse in Example 5.5.

The Fourier transform of *x*(*t*) is

$$X(\omega) = \text{sinc}\left(\frac{\omega}{2\pi}\right)$$

Response of an RC Circuit to a Rectangular Pulse

- The response of the system in the time domain can be found by computing the convolution

$$y(t) = h(t) * x(t)$$

where

$$h(t) = (1/RC)e^{-(1/RC)t}u(t)$$

$$x(t) = \mathrm{rect}(t)$$

THANK YOU