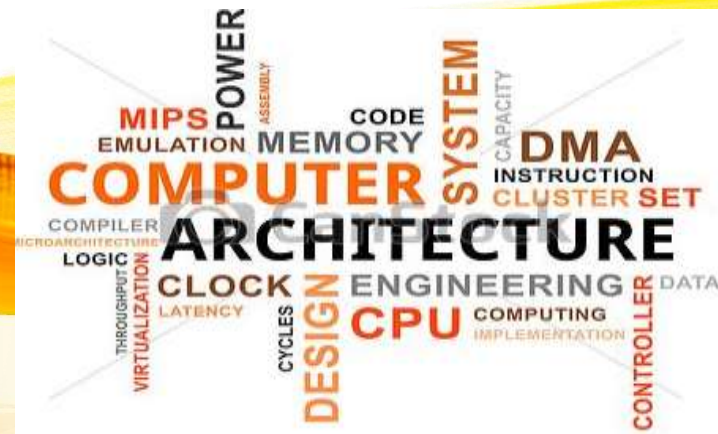


UNIT II

ARITHMETIC OPERATIONS

Addition and subtraction of signed numbers - Design of fast adders - Multiplication of positive numbers - **Signed operand multiplication**- fast multiplication - Integer division - Floating point numbers and operations



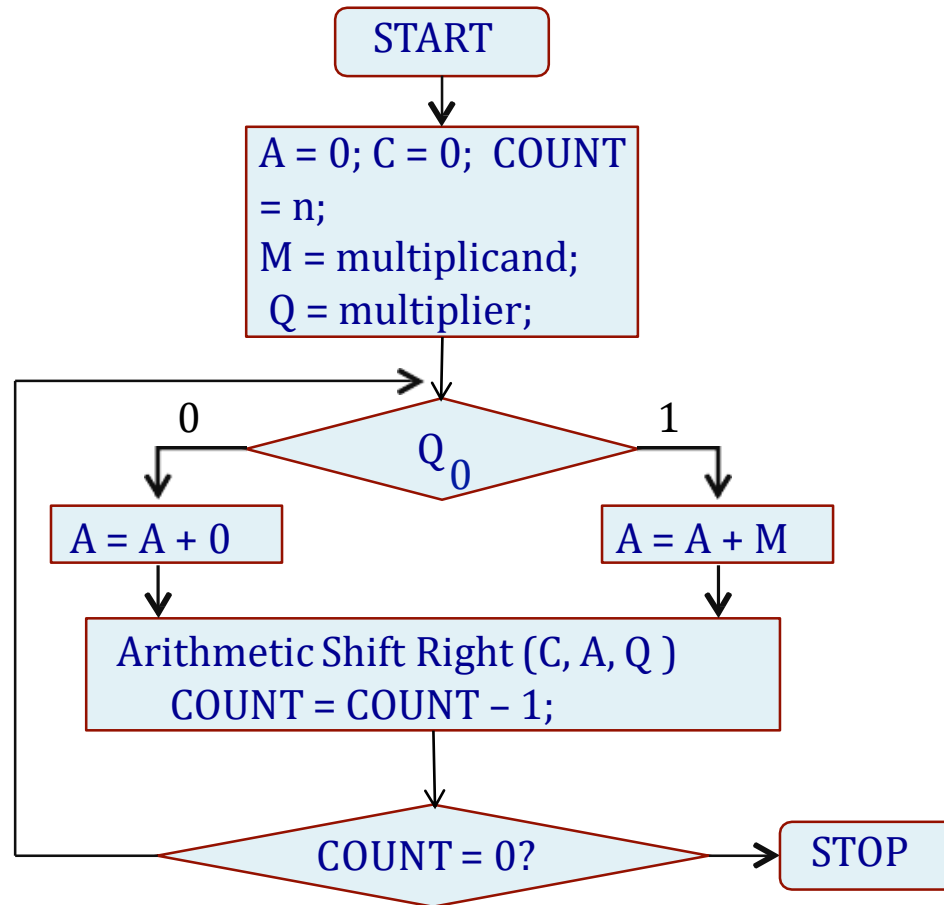


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Recap the previous Class



Unsigned Sequential Multiplication



M: n-bit multiplicand

Q: n-bit multiplier

A: n-bit temporary register

C: 1-bit carry out from adder

Unsigned Sequential Multiplication

Example 1: $(10) \times (13)$

Assume 5-bit numbers.

M: $(01010)_2$

Q: $(01101)_2$

Product = 130

= $(0010000010)_2$

C	A	Q	
0	0 0 0 0 0	0 1 1 0 1	Initialization
0	0 1 0 1 0	0 1 1 0 1	A = A + M Step 1
0	0 0 1 0 1	0 0 1 1 0	Shift
0	0 0 1 0 1	0 0 1 1 0	A = A + 0 Step 2
0	0 0 0 1 0	1 0 0 1 1	Shift
0	0 1 1 0 0	1 0 0 1 1	A = A + M Step 3
0	0 0 1 1 0	0 1 0 0 1	Shift
0	1 0 0 0 0	0 1 0 0 1	A = A + M Step 4
0	0 1 0 0 0	0 0 1 0 0	Shift
0	0 1 0 0 0	0 0 1 0 0	A = A + 0 Step 5
0	0 0 1 0 0	0 0 0 1 0	Shift

Unsigned Sequential Multiplication

Example 2: $(29) \times (21)$

Assume 5-bit numbers.

M: $(11101)_2$

Q: $(10101)_2$

Product = 609

$= (1001100001)_2$

C	A	Q		
0	0 0 0 0 0	1 0 1 0 1	Initialization	
0	1 1 1 0 1	1 0 1 0 1	A = A + M	Step 1
0	0 1 1 1 0	1 1 0 1 0	Shift	
0	0 1 1 1 0	1 1 0 1 0	A = A + 0	Step 2
0	0 0 1 1 1	0 1 1 0 1	Shift	
1	0 0 1 0 0	0 1 1 0 1	A = A + M	Step 3
0	1 0 0 1 0	0 0 1 1 0	Shift	
0	0 1 0 0 1	0 0 0 1 1	A = A + 0	Step 4
1	0 0 1 1 0	0 0 0 1 1	Shift	
0	1 0 0 1 1	0 0 0 0 1	A = A + M	Step 5

Signed Multiplication

- Can extend the basic shift-and-add multiplication method to handle signed numbers.
- One important difference:
 - Required to sign-extend all the partial products before they are added.
 - Recall that for 2's complement representation, sign extension can be done by replicating the sign bit any number of times.

0101 = 0000 0101 = 0000 0000 0000 0101 = 0000 0000 0000 0000 0000 0000 0000 0101

1011 = 1111 1011 = 1111 1111 1111 1011 = 1111 1111 1111 1111 1111 1111 1111 1011

Booth's Algorithm for Signed Multiplication

- In the conventional shift-and-add multiplication as discussed, for n -bit multiplication, we iterate n times.
 - Add either 0 or the multiplicand to the $2n$ -bit partial product (depending on the next bit of the multiplier).
 - Shift the $2n$ -bit partial product to the right.
- Essentially we need ***n additions and n shift operations.***
- Booth's algorithm is an improvement whereby we can avoid the additions whenever consecutive 0's or 1's are detected in the multiplier.
 - Makes the process faster.

Basic Idea Behind Booth's Algorithm

- We inspect two bits of the multiplier (Q_i, Q_{i-1}) at a time.
 - If the bits are same (00 or 11), we only shift the partial product.
 - If the bits are 01, we do an addition and then shift.
 - If the bits are 10, we do a subtraction and then shift.
- Significantly reduces the number of additions / subtractions.
- Inspecting bit pairs as mentioned can also be expressed in terms of *Booth's Encoding*.
 - Use the **symbols +1, -1 and 0** to indicate changes w.r.t. Q_i and Q_{i-1} .
 - 01 \rightarrow +1, 10 \rightarrow -1, 00 or 11 \rightarrow 0.
 - For encoding the least significant bit Q_0 , we assume $Q_{-1} = 0$.

Examples of Booth encoding

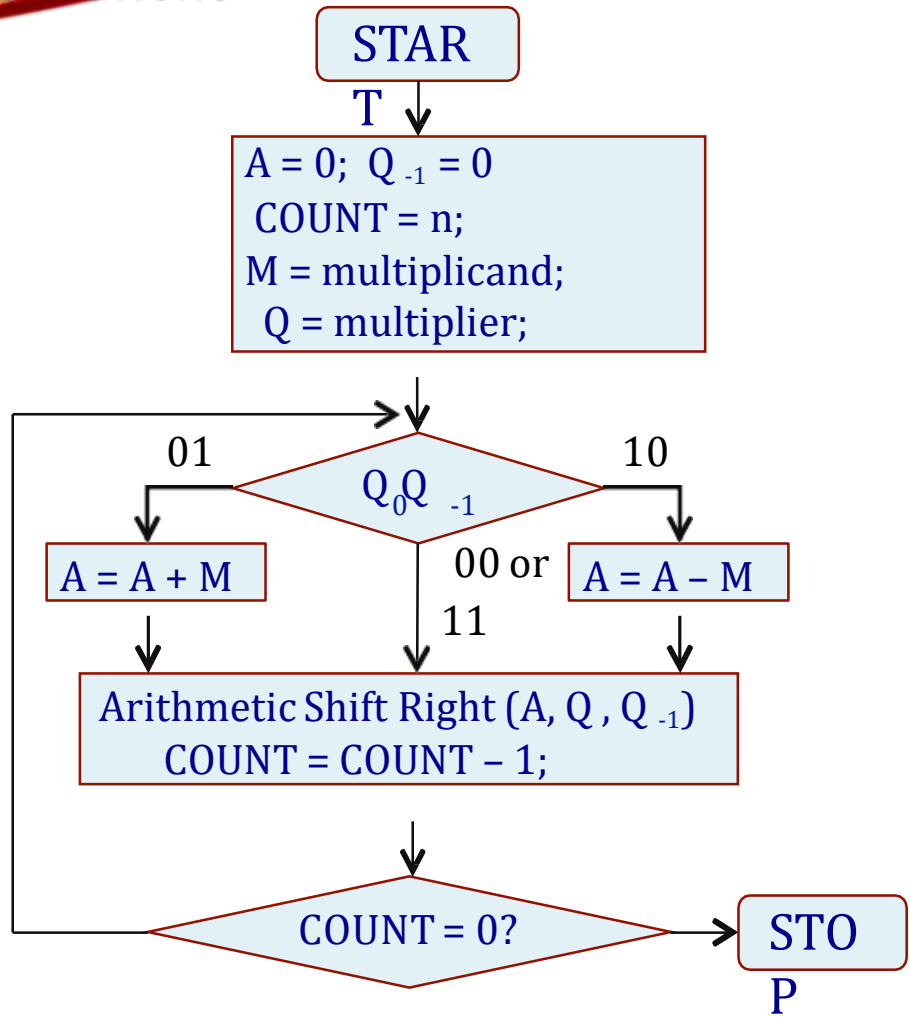
a) 0 1 1 1 0 0 0 0 :: +1 0 0 -1 0 0 0 0

b) 0 1 1 1 0 1 1 0 :: +1 0 0 -1 +1 0 -1 0

c) 0 0 0 0 0 1 1 1 :: 0 0 0 0 +1 0 0 -1

d) 0 1 0 1 0 1 0 1 :: +1 -1 +1 -1 +1 -1 +1 -1

- The last example illustrates the worst case for Booth's multiplication (alternating 0's and 1's in multiplier).
- In the illustrations, we shall show the two multiplier bits explicitly instead of showing the encoded digits.



- M: n-bit multiplicand
- Q: n-bit multiplier
- A: n-bit temporary register
- Q_{-1} : 1-bit flip-flop

Skips over consecutive 0's and 1's of the multiplier Q.



Example 1: $(-10) \times (13)$

Assume 5-bit numbers.

$M: (10110)_2$

$-M: (01010)_2$

$Q: (01101)_2$

Product = -130

$= (110111110)_2$

A	Q	Q ₋₁		
0 0 0 0 0	0 1 1 0	1 0	Initialization	
0 1 0 1 0	0 1 1 0	1 0	A = A - M	Step 1
0 0 1 0 1	0 0 1 1	0 1	Shift	
1 1 0 1 1	0 0 1 1	0 1	A = A + M	Step 2
1 1 1 0 1	1 0 0 1	1 0	Shift	
0 0 1 1 1	1 0 0 1	1 0	A = A - M	Step 3
0 0 0 1 1	1 1 0 0	1 1	Shift	
0 0 0 0 1	1 1 1 1	0 1	Shift	Step 4
1 0 1 1 1	1 1 1 0	0 1	A = A + M	Step 5
1 1 0 1 1	1 1 1 1	0 0	Shift	

Example 2:

$(-31) \times (28)$

Assume 6-bit numbers.

$M: (100001)_2$

$-M: (011111)_2$

$Q: (011100)_2$

Product = -868

$= (110010011100)_2$

A	Q	Q ₋₁
0 0 0 0 0 0	0 1 1	1 0 0 0
0 0 0 0 0 0	0 0 1	1 1 0 0
0 0 0 0 0 0	0 0 0	1 1 1 0
0 1 1 1 1 1	0 0 0	1 1 1 0
0 0 1 1 1 1	1 0 0	0 1 1 1
0 0 0 1 1 1	1 1 0	0 0 1 1
0 0 0 0 1 1	1 1 1	0 0 0 1

Initialization

Shift Step 1

Shift Step 2

$A = A - M$ Step 3

Shift

Shift Step 4

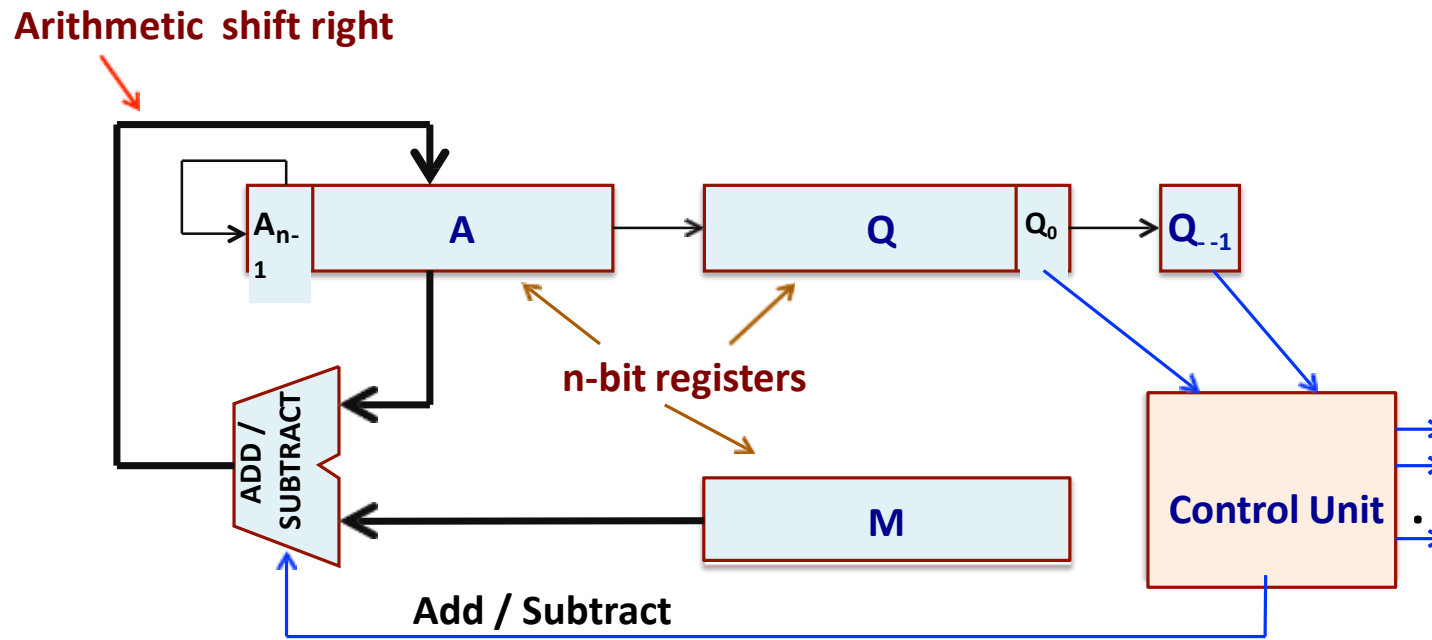
Shift Step 5

$A = A + M$ Step 6

Shift

1 0 0 1 0 0	1 1 1 0 0 0	1
1 1 0 0 1 0	0 1 1 1 0 0	0

Data Path for Booth's Algorithm





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TEXT BOOK

Carl Hamacher, Zvonko Vranesic and Safwat Zaky, “Computer Organization”, McGraw-Hill, 6th Edition 2012.

REFERENCES

1. David A. Patterson and John L. Hennessey, “Computer organization and design”, MorganKauffman ,Elsevier, 5th edition, 2014.
2. William Stallings, “Computer Organization and Architecture designing for Performance”, Pearson Education 8th Edition, 2010
3. John P.Hayes, “Computer Architecture and Organization”, McGraw Hill, 3rd Edition, 2002
4. M. Morris R. Mano “Computer System Architecture” 3rd Edition 2007
5. David A. Patterson “Computer Architecture: A Quantitative Approach”, Morgan Kaufmann; 5th edition 2011

THANK YOU