



## DEPARTMENT OF MATHEMATICS

### UNIT-I FOURIER SERIES

CHANGE OF INTERVAL :

Fourier Series Expansion in  $(0, \infty)$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi}{l} x + \sum_{n=1}^{\infty} b_n \sin \frac{n\pi}{l} x$$

$$\text{where } a_0 = \frac{1}{l} \int_c^{c+2l} f(x) dx$$

$$a_n = \frac{1}{l} \int_c^{c+2l} f(x) \cos \frac{n\pi}{l} x dx$$

$$b_n = \frac{1}{l} \int_c^{c+2l} f(x) \sin \frac{n\pi}{l} x dx$$

1) Find the Fourier series expansion of period  $2l$  for the function  $f(x) = (l-x)^2$  in  $(0, 2l)$

Soln:

$$\text{let } f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi}{l} x + \sum_{n=1}^{\infty} b_n \sin \frac{n\pi}{l} x$$



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$$\begin{aligned}
 \text{Now } a_0 &= \frac{1}{l} \int_0^{2l} (l-x)^2 dx \\
 &= \frac{1}{l} \left[ \frac{(l-x)^3}{-3} \right]_0^{2l} = \frac{1}{l} \left[ \frac{-l^3 - l^3}{-3} \right] \\
 &= \frac{2l^2}{3} \\
 a_n &= \frac{1}{l} \int_0^{2l} (l-x)^2 \cos \frac{n\pi x}{l} dx \\
 &= \frac{1}{l} \left[ (l-x)^2 \frac{\sin \frac{n\pi x}{l}}{\frac{n\pi}{l}} - 2(l-x)(-1) \left( -\frac{\cos \frac{n\pi x}{l}}{\left(\frac{n\pi}{l}\right)^2} \right) + 2 \left( -\frac{\sin \frac{n\pi x}{l}}{\left(\frac{n\pi}{l}\right)^3} \right) \right]_0^{2l} \\
 &= \frac{1}{l} \left[ -2(l-x) \frac{\sin \frac{n\pi x}{l}}{\left(\frac{n\pi}{l}\right)^2} \right]_0^{2l} \\
 &= -\frac{2}{l} \cdot \left( \frac{l}{n\pi} \right)^2 \left[ (-l) \cos \left( \frac{n\pi}{l} \right) - (l) \cos 0 \right] \\
 &= -\frac{2}{l} \times \frac{l^2}{(n\pi)^2} [-l - l] \\
 &= -\frac{4l^2}{n^2\pi^2}
 \end{aligned}$$

$$\begin{aligned}
 b_n &= \frac{1}{l} \int_0^{2l} (l-x)^2 \sin \left( \frac{n\pi x}{l} \right) dx \\
 &= \frac{1}{l} \left[ (l-x)^2 \left( -\frac{\cos \left( \frac{n\pi x}{l} \right)}{\left( \frac{n\pi}{l} \right)} \right) - 2(l-x)(-1) \left( -\frac{\sin \left( \frac{n\pi x}{l} \right)}{\left( \frac{n\pi}{l} \right)^2} \right) + 2 \left( \frac{\cos \left( \frac{n\pi x}{l} \right)}{\left( \frac{n\pi}{l} \right)^3} \right) \right]_0^{2l}
 \end{aligned}$$



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$$= \frac{1}{l} \left[ -l^2 \frac{\cos n\pi}{(n\pi/l)} + 2 \frac{\cos 2nl}{(n\pi/l)^3} - \left( l^2 \frac{\cos 0}{(n\pi/l)} + 2 \frac{\cos 0}{(n\pi/l)^3} \right) \right]$$

$$= \frac{1}{l} \left[ -l^2 \cdot \frac{l}{n\pi} + 2 \frac{l^3}{(n\pi)^3} + l^2 \cdot \frac{l}{n\pi} - 2 \cdot \frac{l^3}{(n\pi)^3} \right]$$

$$= \frac{1}{l} [0]$$

$$= 0$$

$$\therefore f(x) = \frac{l^2}{3} + 4 \frac{l^2}{\pi^2} \sum_{n=1}^{\infty} \frac{\cos(n\pi/l) n}{n^2}$$

Q) Obtain the Fourier series expansion for  $f(x) = \begin{cases} x, & 0 < x < 1 \\ 1-x, & 1 < x < 2 \end{cases}$

Soln:

Here the interval is  $(0, 2)$ .

$$\text{Length} = b-a$$

$$2l = 2 \Rightarrow l = 1$$



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$$\therefore f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx.$$

$$\begin{aligned} \text{Now } a_0 &= \frac{1}{\pi} \int_0^{\pi} f(x) dx \\ &= \int_0^1 x dx + \int_1^{\pi} (1-x) dx \\ &= \left[ \frac{x^2}{2} \right]_0^1 + \left[ x - \frac{x^2}{2} \right]_1^{\pi} \\ &= \frac{1}{2} + \left[ \left( -\frac{1}{2} \right) - \left( 1 - \frac{1}{2} \right) \right] \\ &= \frac{1}{2} + [0 - \frac{1}{2}] \\ &= 0 \end{aligned}$$

$$\begin{aligned} a_n &= \frac{1}{\pi} \int_0^{\pi} f(x) \cos nx dx \\ &= \int_0^1 x \cos nx dx + \int_1^{\pi} (1-x) \cos nx dx \\ &= \left[ x \frac{\sin nx}{n} \Big|_0^1 - \int_0^1 \frac{\sin nx}{n} dx \right] + \\ &\quad \left[ (1-x) \frac{\sin nx}{n} \Big|_1^{\pi} - \int_1^{\pi} (-1) \frac{\sin nx}{n} dx \right], \end{aligned}$$



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$$= \left[ \frac{\cos n\pi}{(n\pi)^2} - \frac{\cos 0}{(n\pi)^2} \right] + \left[ -\frac{\cos 2n\pi}{(n\pi)^2} - \left( -\frac{\cos 0}{(n\pi)^2} \right) \right]$$

$$= \frac{(-1)^n}{(n\pi)^2} - \frac{1}{(n\pi)^2} - \frac{1}{(n\pi)^2} + \frac{(-1)^n}{(n\pi)^2}$$

$$= [(-1)^n - 1] \frac{2}{(n\pi)^2}$$

$$= 2 \left[ \frac{(-1)^n - 1}{n^2\pi^2} \right]$$

$$b_n = \frac{1}{1} \int_0^2 f(x) \sin nx dx$$

$$= \int_0^1 x \sin nx dx + \int_1^2 (1-x) \sin nx dx.$$

$$= \left[ x \left( -\frac{\cos nx}{n\pi} \right) - 1 \left( -\frac{\sin nx}{n\pi} \right) \right]_0^1 + \left[ (1-x) \left( -\frac{\cos nx}{n\pi} \right) - (-1) \left( -\frac{\sin nx}{n\pi} \right) \right]_1^2$$

$$= \left[ -\frac{\cos n\pi}{n\pi} - 0 \right] + \left[ (-1) - \frac{\cos 2n\pi}{n\pi} - 0 \right]$$

$$= -\frac{(-1)^n}{n\pi} + \frac{1}{n\pi}$$

$$= \frac{1 - (-1)^n}{n\pi}$$

$$\therefore f(x) = \sum_{n=1}^{\infty} 2 \left[ \frac{(-1)^n - 1}{n^2\pi^2} \right] \cos n\pi x + \sum_{n=1}^{\infty} \left[ \frac{1 - (-1)^n}{n\pi} \right] \sin n\pi x.$$