Fluid Mechanics and Machineries- Energy equation

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Dynamics of Fluid Flow.

This chapter includes the study of forces causing fluid flow. Thus dynamics of fluid flow is the study of fluid motion with the forces causing flow.

It is analysed by Newton's Second Law of Motion.

Equation of Motion:

According to Newton's Second Law of motion the net force F_x acting on a fluid element in the direction of x is equal to mass m of the fluid element multiplied by the acceleration a_x in the x-direction.

Thus Mathematically

$$F_x = m \cdot a_x \quad \text{---} \ (1)$$

In the fluid flow, the following forces are Present

(i) F_g - gravity force

(ii) F_p - Pressure force

(iii) F_v - Force due to viscosity

(iv) F_t - Force due to turbulence

(v) F_c - Force due to Compressibility

Thus in equation (1) the net force

$$F_x = (F_g)_x + (F_p)_x + (F_v)_x + (F_t)_x + (F_c)_x$$

(i) If the force due to Compressibility F_c is negligible the resulting net force

$$F_x = (F_g)_x + (F_p)_x + (F_v)_x + (F_t)_x + \cancel{F_c}_x$$

and equation of motion are Called Reynolds' equation of motion.

(ii) For flow, where (F_t) is negligible, the resulting equations of motion are known as Navier – Stokes Equation.

$$F_x = F_{g_x} + F_{p_x} + F_{v_x} + \cancel{F_t} + \cancel{F_{c_x}}$$

(iii) If the flow is assumed to be ideal, viscous force (F_v) is zero and equation of motion are known as Eulers Equation of motion.

$$F_x = F_{g_x} + F_{p_x} + \cancel{F_{v_x}} + \cancel{F_{t_x}} + \cancel{F_{c_x}}$$

EULER'S EQUATION OF MOTION :- Energy Equation:-

This is equation of motion in which the forces due to gravity and Pressure are taken into Consideration.

This is derived by Considering the motion of a fluid element along a stream line – as

Consider a stream-line in which flow is taking place in s-direction: as shown in Fig. Consider a cylindrical element of Cross-Section dA and length ds. The forces acting on the cylindrical element are (i) Pressure force $p\,dA$ in the direction of flow

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2. Pressure force $\left(p+\frac{\partial p}{\partial s}ds\right)dA$ opposite to the direction of flow.

3. Weight of element $\rho g\, dA\, ds$

Let θ is the angle between the direction of flow and the line of action of the weight of element

The resultant force on the fluid element in the direction of S must be equal to the mass of fluid element x acceleration in the direction S.

$$p\,dA - \left(p+\frac{\partial p}{\partial s}ds\right)dA - \rho g\, dA\, ds \cos\theta$$

$$= \rho\, dA\, ds \times a_s \quad \text{— (I)}$$

where a_s is the acceleration in the direction of S

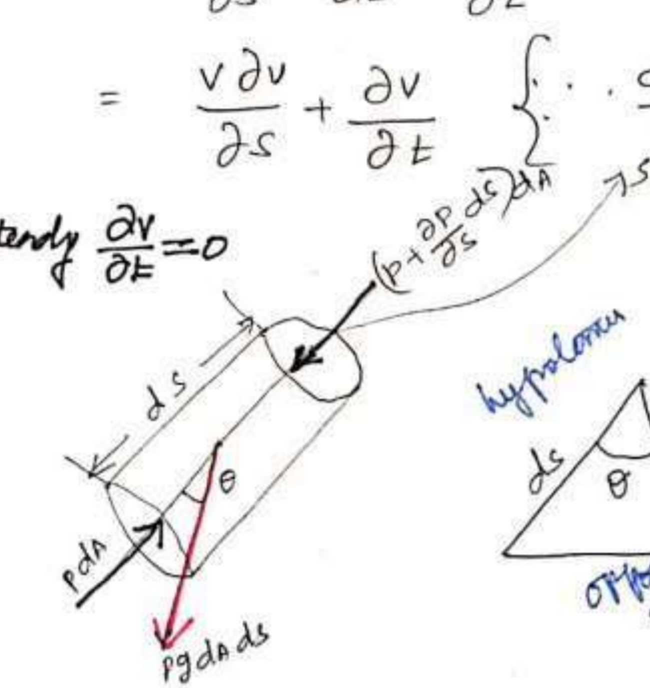
Now $a_s = \frac{dv}{dt}$ where V is a function of S and t

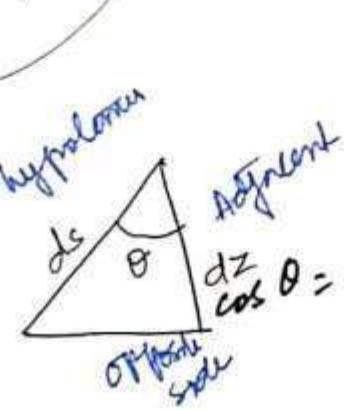
$$= \frac{\partial v}{\partial s}\frac{ds}{dt} + \frac{\partial v}{\partial t}$$

$$= \frac{v\,\partial v}{\partial s} + \frac{\partial v}{\partial t} \quad \left\{\because \frac{ds}{dt} = v\right\}$$

If the flow is steady $\frac{\partial v}{\partial t} = 0$

$$a_s = \frac{v\,\partial v}{\partial s}$$





$\cos\theta = \frac{dz}{ds}$

Substituting the value of a_s in equation ② and simplifying the equation we get

$$-\frac{\partial P}{\partial s}\, ds\, dA - \rho g\, dA\, ds \cos\theta = \rho\, dA\, ds \times v\frac{\partial v}{\partial s}$$

Divided by $\rho\, ds\, dA$, $-\frac{\partial P}{\rho \partial s} - g$

(or) $\frac{\partial P}{\rho \partial s} + g \cos\theta + v\,\frac{\partial v}{\partial s} = 0$

But from figure & Eqn ① we have $\cos\theta = \frac{dz}{ds}$

$$\frac{1}{\rho}\,\frac{dP}{ds} + g\,\frac{dz}{ds} + \frac{v\,dv}{ds} = 0$$

(or)

$$\frac{dP}{\rho} + g\,dz + v\,dV = 0$$

$$\boxed{\frac{dP}{\rho} + g\,dz + v\,dV = 0} \quad \text{——— ③}$$

Eqn ③ is known as Euler's equation of motion

<u>Bernoulli's Equation from Euler's Equation</u>

<u>Assumptions.</u>

(i) The fluid is ideal i.e viscosity is zero [inviscid (i.e) Frictionless].

(ii) The flow is steady

(iii) The flow is incompressible ($\rho = c$)

(iv) The flow is irrotational.

Bernoulli's equation is obtained by integrating the Euler's equation of motion ③

$$\int \frac{dP}{\rho} + \int g\,dz + \int v\,dv = \text{Constant}$$

If flow is incompressible ρ is Constant

$$\frac{P}{\rho} + gz + \frac{V^2}{2} = \text{Constant}$$

$$\frac{P}{\rho g} + Z + \frac{V^2}{2g} = \text{constant}$$

$$\frac{P}{\rho g} + \frac{V^2}{2g} + Z = \text{Constant} \quad \text{—} (4)$$

Equation (4) is Bernoulli's equation.

$\frac{P}{\rho g}$ = Pressure energy per unit weight of fluid or pressure head

$\frac{V^2}{2g}$ = Kinetic energy per unit weight or Kinetic head

Z = Potential energy per unit weight (or) Potential head.

Applications: 1. Venturimeter 2. Orifice meter 3. Pitot Tube.

Ans: State Bernoulli's theorem It states that in a steady, ideal flow of an incompressible fluid, the total energy at any point of the fluid is Constant. The total energy Consists of Pressure energy, KE and Potential energy or datum energy. These energies per Unit weight of the fluid are

Bernoulli's equation for Real Fluid:

The Bernoulli's equation was derived on the assumption that fluid is inviscid (non-viscous) and therefore frictionless.

But all the real fluids are viscous and hence offer resistance to flow. Thus there are always some losses in fluid flows and hence in the application of Bernoulli's equation these losses have to be taken into consideration. Thus the Bernoulli's equation for real fluids between points 1 and 2 given as

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_L$$

h_L - Loss of energy between points 1 and 2.