DEPARTMENT OF MECHANICAL ENGINEERING

19GET275 & VQAR –I

Unit I – Quatitative Ability I

Number Theory

Number theory is a branch of mathematics that deals with the properties and relationships of numbers, especially integers. It is one of the oldest and most fundamental areas of mathematics, with roots dating back to ancient civilizations. Number theory explores various topics related to integers, such as divisibility, prime numbers, congruence, and Diophantine equations.

Some key concepts and areas within number theory:

1. **Divisibility**: Number theory begins with the concept of divisibility, where one integer can be divided by another without leaving a remainder. The fundamental theorem of arithmetic states that every positive integer can be uniquely factorized into prime numbers.

2. **Prime Numbers**: Prime numbers are natural numbers greater than 1 that have only two distinct positive divisors: 1 and themselves. They play a central role in number theory. The distribution of prime numbers and the existence of infinitely many primes are well-known conjectures in number theory.

3. **Congruences: Congruences** involve relationships between integers that have the same remainder when divided by a fixed integer. Modular arithmetic is a key tool in number theory and is used to study congruences.

4. **Diophantine Equations**: Diophantine equations are equations that involve only integer solutions. A famous example is Fermat's Last Theorem, which states that there are no three positive integers a, b, and c that satisfy the equation $a^n + b^n = c^n$ for any integer value of n greater than 2.

5. **Number Theoretic Functions**: Number theory also deals with functions like the Euler's totient function, which counts the number of positive integers less than a given integer that are coprime to it. Other functions include the divisor function and the Möbius function.
6. The Riemann Hypothesis: The Riemann Hypothesis is one of the most famous unsolved problems in mathematics and is closely related to the distribution of prime numbers. It posits that all non-trivial zeros of the Riemann zeta function have a specific form.
7. Analytic Number Theory: This branch of number theory combines techniques from complex analysis with number theory to study the distribution of prime numbers and related topics.
8. Algebraic Number Theory: Algebraic number theory extends number theory by introducing concepts from abstract algebra, particularly the study of number fields and algebraic integers.
9. Computational Number Theory: With the advent of computers, computational methods have become essential in number theory for solving problems, conducting searches for large primes, and verifying conjectures.

Divisibility Rules

Rule for 2 - A number is divisible by 2 when the number ends with 0,2,4,6,8.

Rule for 3 - If the sum of digit is divisible by 3 than the number is divisible by 3.

Rule for 4 - If the last two digit is divisible by 4, than the number is divisible by 4.

Rule for 5 - Number is divisible by 5 if the last digit is 0 or 5.

Rule for 6 - A number is divisible by if number is divisible by 2 and 3.

Rule for 7 - Double the last digit and subtract it from the remaining leading truncated number to check if the result is divisible by 7 until no further division is possible

Rule for 8 - If the last 3 number is divisible by 8 than the number is divisible by 8.

Rule for 9 - Same as divisibility of 3 but sum of digits is divided by 9 in place of 3

•**Rule for 10 -** If the last digit is 0 than the number is divisible by 10.

UNIT I
QUANTITATIVE ABILITY I

Number theory- Shortcuts, Divisibility rule- Unit place deduction-LCM &HCF, Square root and Cube Root, Decimal & Fraction Percentage, Profit, loss and discount, Simple and compound interest, Ratio & Proportions, Mixtures & Allegation, Partnership.

Number Theory

- **Number System** – represent in different forms
- Hindu-Arabic system - 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9 – **Digits**

Insignificant digit	Significant digits.
0	1 to 9

Numerals

Cardinal Numbers		Ordinal Numbers	
1	one	1st	first
2	two	2nd	second
3	three	3rd	third
4	four	4th	fourth
5	five	5th	fifth
6	six	6th	sixth
7	seven	7th	seventh
8	eight	8th	eighth
9	nine	9th	ninth
10	ten	10th	tenth
11	eleven	11th	eleventh
12	twelve	12th	twelfth
13	thirteen	13th	thirteenth
14	fourteen	14th	fourteenth
15	fifteen	15th	fifteenth
16	sixteen	16th	sixteenth
17	seventeen	17th	seventeenth
18	eighteen	18th	eighteenth
19	nineteen	19th	nineteenth
20	twenty	20th	twentieth

1	I
2	II
3	III
4	IV
5	V
6	VI
7	VII
8	VIII
9	IX
10	X

11	XI
20	XX
30	XXX
40	XL
50	L
60	LX
70	LXX
80	LXXX
90	XC
100	C

200	CC
300	CCC
400	CD
500	D
600	DC
700	DCC
800	DCCC
900	CM
1000	M
1001	MI

I	V	X	L	C	D	M
1	5	10	50	100	500	1000

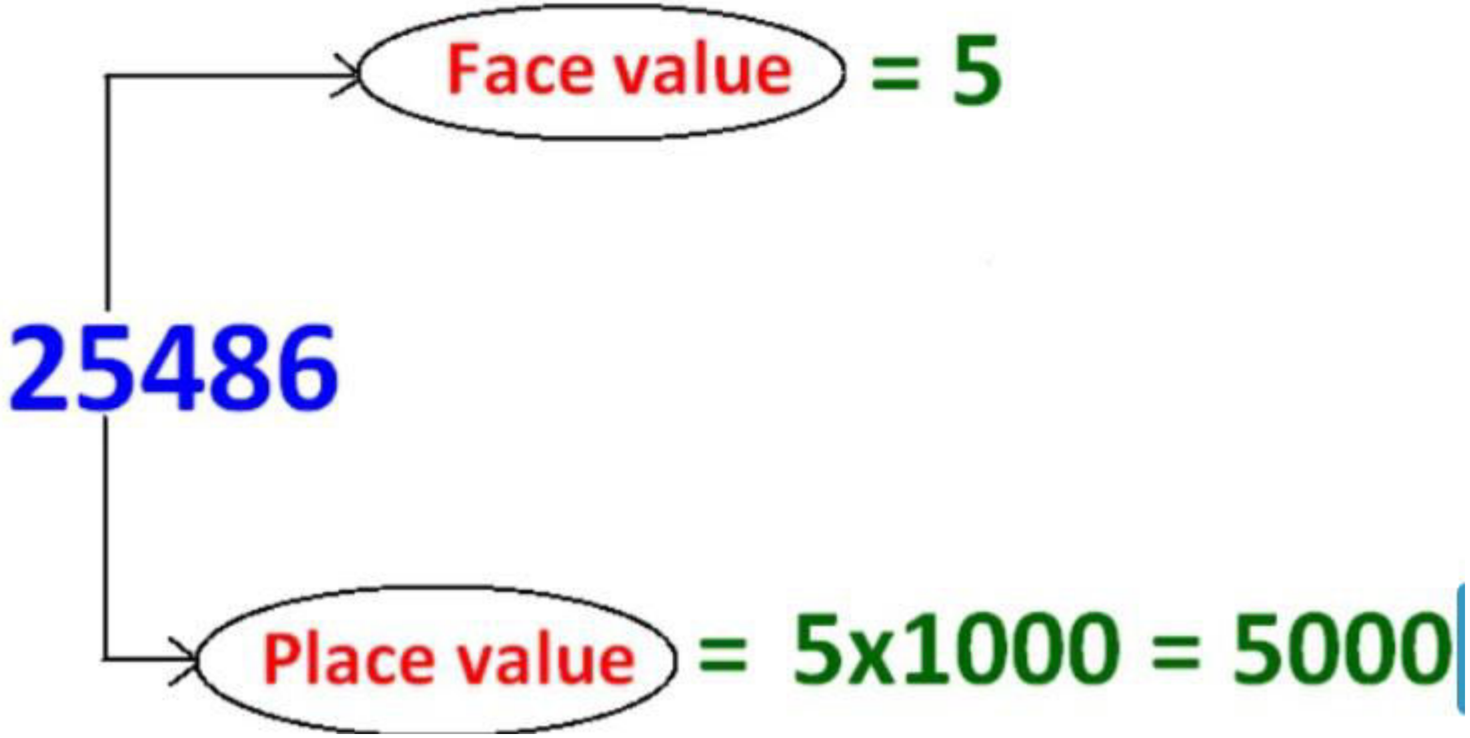
How to Write a Number?

- Right to left

Indian Place Value System								
CRORES		LAKHS		THOUSANDS		ONES		
TC	C	TL	L	T-TH	TH	H	T	O
		2	3	1	9	6	1	7

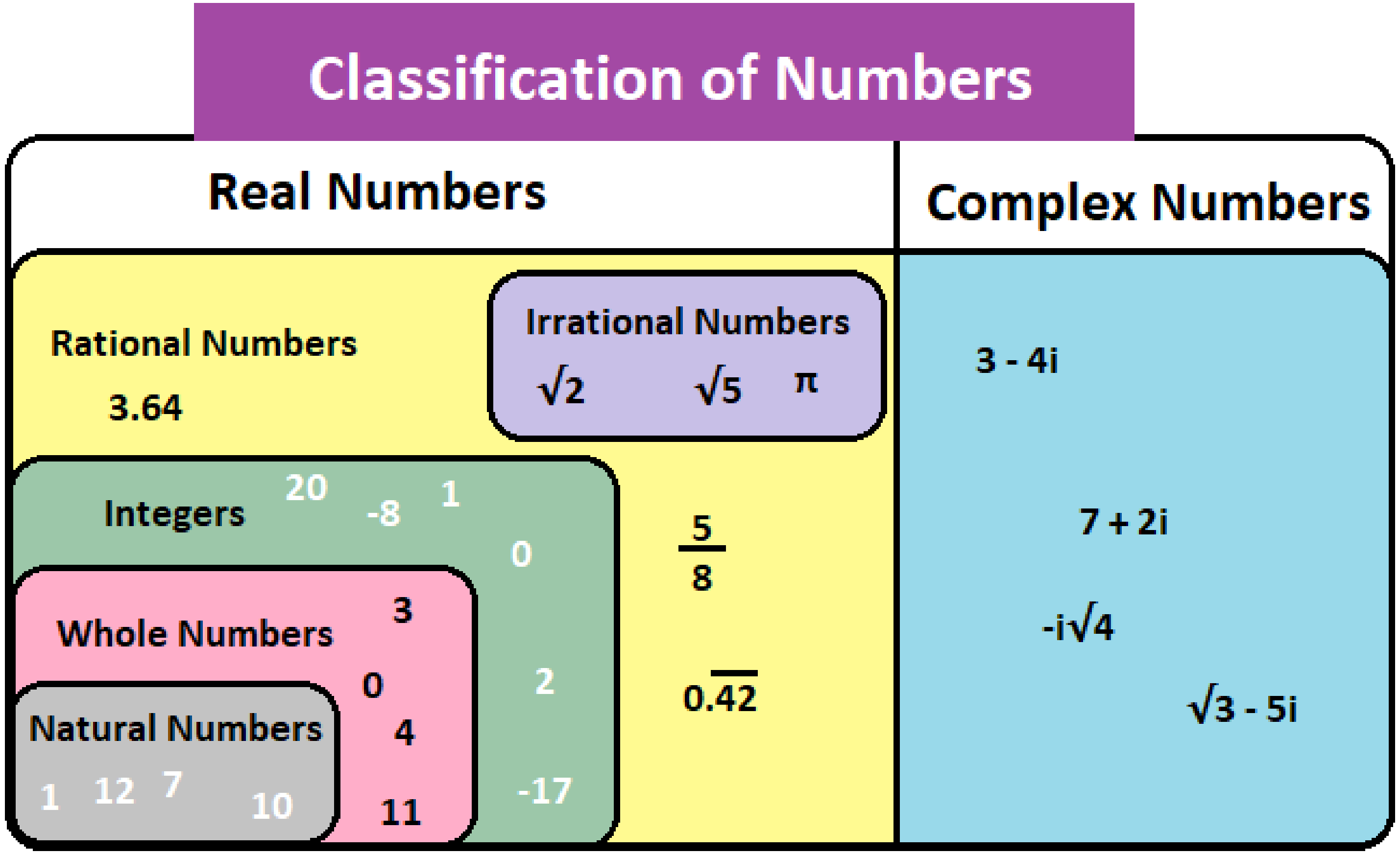
International Place Value Chart								
MILLIONS			THOUSANDS			ONES		
HM	TM	M	HTh	TTh	Th	H	T	O
		2	3	1	9	6	1	7

Face Value and Place Value of the Digits in a Number



Dividend = (Divisor * Quotient)+ Remainder

Types of Numbers



Even Numbers
Odd Numbers
Prime Numbers – 1 and itself
Composite - non-prime natural number
Coprimes - Two natural numbers & HCF is 1

Addition	+
Subtraction	-
Multiplication	x
Division	÷

Number Theory and Shortcuts

1.Sum of natural numbers from 1 to n

$$\frac{n(n+1)}{2}$$

e.g Sum of natural numbers from 1 to 40 = 40(40+1)/2 = 820

2. Sum of squares of first n natural numbers is =

$$\frac{n(n+1)(2n+1)}{6}$$

3. Sum of the squares of first n even natural numbers is

$$\frac{2}{3}n(n+1)(2n+1)$$

Number Theory and Shortcuts

4. Sum of cubes of first n natural numbers is

$$\left[\frac{n(n+1)}{2}\right]^2$$

5. Any number N can be represented in the decimal system of number as

$$N = n_k 10^k + n_{k-1} 10^{k-1} + n \ldots + n_i 10 + n_0$$

Divisibility Rules

Rule for 2 - A number is divisible by 2 when the number ends with 0,2,4,6,8.

Rule for 3 - If the sum of digit is divisible by 3 than the number is divisible by 3.

Rule for 4 - If the last two digit is divisible by 4, than the number is divisible by 4.

Rule for 5 - Number is divisible by 5 if the last digit is 0 or 5.

Divisibility Rules

- **Rule for 6 -** A number is divisible by if number is divisible by 2 **AND** 3.
- **Rule for 7 -** Double the last digit and subtract it from the remaining leading truncated number to check if the result is divisible by 7 until no further division is possible
- **Rule for 8 -** If the last 3 number is divisible by 8 than the number is divisible by 8.
- **Rule for 9 -** Same as divisibility of 3 but sum of digits is divide by 9 in place of 3
- **Rule for 10 -** If the last digit is 0 than the number is divisible by 10.

Formulas

1. $(a + b)(a - b) = (a^2 - b^2)$
2. $(a + b)^2 = (a^2 + b^2 + 2ab)$
3. $(a - b)^2 = (a^2 + b^2 - 2ab)$
4. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$
5. $(a^3 + b^3) = (a + b)(a^2 - ab + b^2)$
6. $(a^3 - b^3) = (a - b)(a^2 + ab + b^2)$
7. $(a^3 + b^3 + c^3 - 3abc) = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ac)$
8. when $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$

https://www.indiabix.com/aptitude/numbers/001002


Thank You

Number System Problems

1. Which one of the following is not a prime number?

   A. 31

   B. 61

   C. 71

   D. 91

   Answer: Option **D**

   Explanation:

   91 is divisible by 7. So, it is not a prime number.

2. $(112 \times 5^4) = ?$

   A. 67000

   B. 70000

   C. 76500

   D. 77200

   Answer: Option **B**

   Explanation:

   $$(112 \times 5^4) = 112 \times \left(\frac{10}{2}\right)^4 = \frac{112 \times 10^4}{2^4} = \frac{1120000}{16} = 70000$$

$1397 \times 1397 = ?$

A. 1951609

B. 1981709

C. 18362619

D. 2031719

E. None of these

Answer: Option **A**

Explanation:

$$\begin{aligned} 1397 \times 1397 &= (1397)^2 \\ &= (1400 - 3)^2 \\ &= (1400)^2 + (3)^2 - (2 \times 1400 \times 3) \\ &= 1960000 + 9 - 8400 \\ &= 1960009 - 8400 \\ &= 1951609. \end{aligned}$$

$(935421 \times 625) = ?$

A. 575648125

B. 584638125

C. 584649125

D. 585628125

Answer: Option **B**

Explanation:

$$935421 \times 625 = 935421 \times 5^4 = 935421 \times \left(\frac{10}{2}\right)^4$$

$$= \frac{935421 \times 10^4}{2^4} = \frac{9354210000}{16}$$

$$= 584638125$$

What least number must be added to 1056, so that the sum is completely divisible by 23 ?

A. 2

B. 3

C. 18

D. 21

E. None of these

Answer: Option **A**

Explanation:

```
23) 1056 (45
     92
     ---
     136
     115
     ---
      21
     ---

Required number = (23 - 21)
                = 2.
```

How many of the following numbers are divisible by 132 ?
264, 396, 462, 792, 968, 2178, 5184, 6336

A. 4

B. 5

C. 6

D. 7

Answer: Option **A**

132 = 4 x 3 x 11

So, if the number divisible by all the three number 4, 3 and 11, then the number is divisible by 132 also.

264 $\rightarrow$ 11,3,4 (/)

396 $\rightarrow$ 11,3,4 (/)

462 $\rightarrow$ 11,3 (X)

792 $\rightarrow$ 11,3,4 (/)

968 $\rightarrow$ 11,4 (X)

2178 $\rightarrow$ 11,3 (X)

5184 $\rightarrow$ 3,4 (X)

6336 $\rightarrow$ 11,3,4 (/)

Therefore the following numbers are divisible by 132 : 264, 396, 792 and 6336.

Required number of number = 4.

The largest 4 digit number exactly divisible by 88 is:

A. 9944

B. 9768

C. 9988

D. 8888

E. None of these

Answer: Option **A**

Explanation:

```
Largest 4-digit number = 9999


 88) 9999 (113
     88
     ----
     119
      88
     ----
      319
      264
      ---
       55
      ---

 Required number = (9999 - 55)
```

Type 1:Find the largest or smallest number

Question 1. Find the smallest 4 digit number which is exactly divisible by 41?

Options.

A. 1000

B. 1023

C. 1025

D. 1012

Solution Smallest 4 digit number is 1000

On dividing 1000 by 41, remainder = 16

Required number = 1000 + (41 – 16) = 1025

Correct option: C

Question 2. Find the Largest 3-digit number which is exactly divisible by 25?

Options.

A. 975

B. 905

C. 980

D. 950

Solution Largest Three digit numbers is 999

On dividing 999 by 25, remainder = 24

Required number = 999 – 24 = 975

Correct option: A

Type 2:Which of the following numbers is/or not divisible by given number.

Question 1. Which of these numbers is divisible by 3?

Options.

A. 1003

B. 253

C. 1031

D. 1221

Solution 1003 = 1 + 0 + 0 + 3 = 4, 4 is not divisible by 3

253 = 2 + 5 + 3 = 10, 10 is not divisible by 3

1031 = 1 + 0 + 3 + 1 = 5, 5 is not divisible by 3

1221 = 1 + 2 + 2 + 1= 6, 6 is divisible by 3

Correct option: D

Question 2. Which of these numbers is not divisible by 10?

Options.

A. 1250

B. 1253

C. 1930

D. 1220

Solution Last digit of 1253 is not 0 so it is not divisible by 10

Correct option: B

Type 3: Tips and Tricks to Solve Divisibility Questions.

Find the remainder

Question 1. **Find out the remainder of \frac{2^{12}}{5}5212**

Options.

A. 1

B. 2

C. 0

D. 3

Solution Convert 2^{12}212 in multiple of 16 = 16 x 16 x 16

=2^{4}\times 2^{4}\times 2^{4}24×24×24

Now divide each number by 5

On dividing 16 by 5 we get remainder as 1

Now, multiply all the remainders 1 x 1 x 1 = 1

Correct option: A

Question 2. **Find out the remainder when 7^{4}74 is divided by 5.**

Options.

A. 0

B. 4

C. 1

D. 2

Solution Divide 7 by 5 Remainder will 2

2\times 2\times 2\times 22×2×2×2 = 16

Now divide 16 by 5

On dividing 16 by 5 we get remainder as 1

Correct option: C