



## DEPARTMENT OF MATHEMATICS

### UNIT-II FOURIER TRANSFORM

Find the sine transform of the function  $f(x) = \frac{e^{-ax}}{x}$ .

Soln:

$$\begin{aligned} \text{WKT } F_s(s) &= F_s[f(x)] = \sqrt{\frac{2}{\pi}} \int_0^{\infty} f(x) \sin sx \, dx \\ &= \sqrt{\frac{2}{\pi}} \int_0^{\infty} \frac{e^{-ax}}{x} \sin sx \, dx. \end{aligned}$$

D.w.r. to 's' ODS we get,

$$\begin{aligned} \frac{d}{ds} [F_s(s)] &= \sqrt{\frac{2}{\pi}} \int_0^{\infty} \frac{d}{ds} \left[ \frac{e^{-ax}}{x} \sin sx \right] dx \\ &= \sqrt{\frac{2}{\pi}} \int_0^{\infty} \frac{e^{-ax}}{x} \frac{\cos sx}{s} \cdot x \, dx \\ &= \sqrt{\frac{2}{\pi}} \int_0^{\infty} e^{-ax} \cos sx \, dx \\ &= \sqrt{\frac{2}{\pi}} \left[ \frac{a}{a^2 + s^2} \right] \end{aligned}$$

Integrating w.r. to 's' we get,

$$\begin{aligned} F_s(s) &= \sqrt{\frac{2}{\pi}} \int \frac{a}{a^2 + s^2} ds \\ &= \sqrt{\frac{2}{\pi}} a \int \frac{1}{a^2 + s^2} ds \\ &= \sqrt{\frac{2}{\pi}} a \cdot \frac{1}{a} \tan^{-1} \left( \frac{s}{a} \right) + c \\ &= \sqrt{\frac{2}{\pi}} \tan^{-1} \left( \frac{s}{a} \right) + c \end{aligned}$$



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### UNIT-II FOURIER TRANSFORM

Q) Find the cosine transform of the function  $f(x) = \frac{e^{-ax}}{x}$ .

Soln: Wkt  $F_c(s) = F_c\{f(x)\} = \sqrt{\frac{2}{\pi}} \int_0^{\infty} f(x) \cos sx \, dx$   
 $= \sqrt{\frac{2}{\pi}} \int_0^{\infty} \frac{e^{-ax}}{x} \cos sx \, dx$

D.w.r. to 's' O.B.S we get,

$$\frac{d}{ds} [F_c(s)] = \sqrt{\frac{2}{\pi}} \int_0^{\infty} \frac{d}{ds} \left[ \frac{e^{-ax}}{x} \cdot \cos sx \right] dx$$

$$= \sqrt{\frac{2}{\pi}} \int_0^{\infty} \frac{e^{-ax}}{x} \cdot -\sin sx \, dx$$

$$= -\sqrt{\frac{2}{\pi}} \int_0^{\infty} e^{-ax} \sin sx \, dx$$

$$\frac{d}{ds} [F_c(s)] = -\sqrt{\frac{2}{\pi}} \left[ \frac{s}{s^2 + a^2} \right]$$

Integrating w.r. to 's' we get

$$F_c(s) = -\sqrt{\frac{2}{\pi}} \int \frac{s}{s^2 + a^2} ds$$

$$= -\sqrt{\frac{2}{\pi}} \cdot \frac{1}{2} \log(s^2 + a^2)$$

$$= -\frac{1}{\sqrt{2\pi}} \log(s^2 + a^2)$$