



## DEPARTMENT OF MATHEMATICS

### UNIT-II FOURIER TRANSFORM

3) Using Parseval's Identity calculate  $\int_0^\infty \frac{n^2}{(a^2+n^2)^2} dn$ , if  $a =$

soln:

$$\text{WKT } F_s(s) = f_s [f(m)] = \sqrt{\frac{2}{\pi}} \int_0^\infty e^{-am} m \sin m s n d m = \sqrt{\frac{2}{\pi}} \left[ \frac{s}{a^2 + s^2} \right]$$

Parseval's Identity:

$$\int_0^\infty (f(m))^2 dm = \int_0^\infty [f_s(s)]^2 ds$$

$$\text{Here } f(m) = e^{-am}$$

$$\int_0^\infty (e^{-am})^2 dm = \int_0^\infty \left[ \sqrt{\frac{2}{\pi}} \left[ \frac{s}{a^2 + s^2} \right] \right]^2 ds$$

$$\int_0^\infty e^{-2am} dm = \frac{2}{\pi} \int_0^\infty \left( \frac{s}{a^2 + s^2} \right)^2 ds$$

$$\frac{e^{-2am}}{-2a} \Big|_0^\infty = \frac{2}{\pi} \int_0^\infty \left( \frac{s}{a^2 + s^2} \right)^2 ds$$

$$\frac{1}{2a} \cdot \frac{\pi}{2} = \int_0^\infty \left( \frac{s}{a^2 + s^2} \right)^2 ds$$

$$\text{put } s = n$$

$$\frac{\pi}{4a} = \int_0^\infty \left[ \frac{n}{a^2 + n^2} \right]^2 dn$$



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Q) Evaluate  $\int_0^\infty \frac{n^2}{(n^2+a^2)(n^2+b^2)} dn$  using transforms.

$$\text{Wk} \quad f_s(s) = F_s[f(n)] = \sqrt{\frac{2}{\pi}} \int_0^\infty e^{-an} f(n) \sin sn dn = \sqrt{\frac{2}{\pi}} \left[ \frac{s}{a^2+s^2} \right]$$

$$G_s(s) = F_s[g(n)] = \sqrt{\frac{2}{\pi}} \int_0^\infty e^{-bn} g(n) \sin sn dn = \sqrt{\frac{2}{\pi}} \left[ \frac{s}{b^2+s^2} \right]$$

Parseval's Identity:

$$\text{Hence } f(n) = e^{-an}, g(n) = e^{-bn} \quad \int_0^\infty f(n) \cdot g(n) dn = \int_0^\infty F_s(s) G_s(s) ds$$

$$\int_0^\infty e^{-an} \cdot e^{-bn} dn = \int_0^\infty F_s(f(n)) G_s(g(n)) ds$$

$$\int_0^\infty e^{-(a+b)n} dn = \int_0^\infty \sqrt{\frac{2}{\pi}} \left[ \frac{s}{s^2+a^2} \right] * \sqrt{\frac{2}{\pi}} \left[ \frac{s}{s^2+b^2} \right] ds$$

$$\left. \frac{e^{-(a+b)n}}{-(a+b)} \right|_0^\infty = \frac{2}{\pi} \int_0^\infty \frac{s^2}{(s^2+a^2)(s^2+b^2)} ds$$

$$\frac{1}{a+b} \cdot \frac{\pi}{2} = \int_0^\infty \frac{s^2}{(s^2+a^2)(s^2+b^2)} ds$$

put  $s=n$

$$\frac{\pi}{2(a+b)} = \int_0^\infty \frac{n^2}{(n^2+a^2)(n^2+b^2)} dn.$$



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Using transforms evaluate:

$$(i) \int_0^\infty \frac{dn}{(n^2 + a^2)^2}$$

$$(ii) \int_0^\infty \frac{ds}{(s^2 + 4)(s^2 + 1)}$$

$$(iii) \int_0^\infty \frac{n^2}{(n^2 + 9)(n^2 + 16)} dn$$