HOMOGENEOUS EQUATIONS :

$$(D^2+2DD'+D'^2)z = \cos(x-y)$$

A.E. is $m^2+2m+1=0$

$\Rightarrow m=-1,\ m=-1$

C.F. is $z = f_1(y-x) + x f_2(y-x)$

$$P.I = \frac{1}{D^2+2DD'+D'^2}\cos(x-y)$$

$$= \frac{1}{-1+2-1=0}\cos(x-y)$$

D.w.r. to 'D' in 'Dr' & multi. by x in the 'Nr'.

$$= x\frac{1}{2D+2D'}x\cos(x-y)$$

$$= \frac{1}{2D+2D'}\times\frac{2D-2D'}{2D-2D'}x\cos(x-y)$$

$\left\{\begin{array}{l} D\to D.w.r.to\ x \\ D'\to D.w.r.to\ y \end{array}\right.$

$$= \frac{2x[D-D']\cos(x-y)}{4D^2-4D'^2}$$

$D^2 = -(1)^2 = -1$

$D'^2 = -(-1)^2 = -1$

$$= \frac{2x[D(\cos(x-y)) - D'\cos(x-y)]}{4(-1)-4(-1)}$$

$[D^2 - DD' - 2D'^2]z = x^2y + (2x+3y)$

AE: $m^2 - m - 2 = 0 \Rightarrow m = 2, -1$

CF: $z = f_1(y-x) + f_2(y+2x)$

P.I$_1$: $= \dfrac{1}{D^2 - DD' - 2D'^2} x^2y$

$$= \frac{1}{D^2\left[1 - \left(\frac{DD' + 2D'^2}{D^2}\right)\right]} x^2y = \frac{1}{D^2}\left[1 - \left(\frac{D'}{D} + \frac{2D'^2}{D^2}\right)\right]^{-1} x^2y$$

$$= \frac{1}{D^2}\left[1 + \frac{D'}{D} + \frac{2D'^2}{D^2}\right] x^2y$$

$$= \frac{1}{D^2}\left[x^2y + \frac{D'}{D}(x^2y) + \frac{2D'^2}{D^2}(x^2y)\right]$$

$$= \frac{1}{D^2}\left[x^2y + \frac{1}{D}\frac{d}{dy}(x^2y) + \frac{2}{D^2}\frac{d^2}{dy^2}(x^2y)\right]$$

$$= \frac{1}{D^2}\left[x^2y + \frac{1}{D}(x^2) + \frac{2}{D^2}(0)\right]$$

$$= \frac{1}{D^2}\left[x^2y + \frac{x^3}{3}\right]$$

$$= \frac{1}{D}\int\left(x^2y + \frac{x^3}{3}\right)dx$$

$$= \int\left(\frac{x^3}{3}y + \frac{x^4}{12}\right)dx$$

$$= \frac{x^4}{12}y + \frac{x^5}{60}$$

$$P.I_2 = \frac{1}{D^2 - DD' - 2D'^2}(2x+3y)$$

$$= \frac{1}{D^2\left[1-\left(\frac{D'}{D}+\frac{2D'^2}{D^2}\right)\right]}(2x+3y)$$

$$= \frac{1}{D^2}\left[1-\left(\frac{D'}{D}+\frac{2D'^2}{D^2}\right)\right]^{-1}(2x+3y)$$

$$= \frac{1}{D^2}\left[1+\frac{D'}{D}+\frac{2D'^2}{D^2}\right](2x+3y)$$

$$= \frac{1}{D^2}\left[2x+3y+\frac{D'}{D}(2x+3y)+\frac{2D'^2}{D^2}(2x+3y)\right]$$

TYPE IV: $RHS = f(x,y) = e^{ax+by}x^m y^n$ (or) $e^{ax+by} \cos(ax+by)$, $\sin(ax+by)$

$$P.I = \frac{1}{\phi(D,D')} e^{ax+by} . x^m y^n .$$

Replace $D \to D+a$; $D' \to D+b$. Then type III rule (or) type II rule.

1) Solve: $(D^2 - 2DD' + D'^2)z = x^2y^2 . e^{x+y}$

Soln: A.E. is $m^2 - 2m + 1 = 0$

$$(m-1)^2 = 0$$

$$m = +1, +1$$

∴ the roots are real and equal.

C.F is $z = f_1(y+mx) + x f_2(y+mx)$

$$= f_1(y+x) + x f_2(y+x)$$

P.I. $\quad P.I = \dfrac{1}{D^2 - 2DD' + D'^2} . x^2y^2 e^{x+y}$

Replace $D \to D+1 \; ; \; D' \longrightarrow D'+1$

$$= \frac{1}{(D+1)^2 - 2(D+1)(D'+1) + (D'+1)^2} . e^{x+y} . x^2y^2$$

$$= \frac{1}{D^2+2D+1-2[DD'+D+D'+1]+D'^2+2D'+1} \cdot e^{x+y} \cdot x^2y^2$$

$$= \frac{1}{D^2+\cancel{2D}+\cancel{1}-2DD'-\cancel{2D}-\cancel{2D'}-\cancel{2}+D'^2+\cancel{2D'}+\cancel{1}} \cdot e^{x+y} \cdot x^2y^2$$

$$= \frac{1}{D^2-2DD'+D'^2} e^{x+y} \cdot x^2y^2$$

$$= e^{x+y} \cdot \frac{1}{D^2}\left[1-\left(\frac{2D'}{D}-\frac{D'^2}{D^2}\right)\right]^{-1} x^2y^2$$

$$= e^{x+y} \cdot \frac{1}{D^2}\left[1+\left(\frac{2D'}{D}-\frac{D'^2}{D^2}\right)\right] x^2y^2$$

$$= e^{x+y} \cdot \frac{1}{D^2}\left[x^2y^2+\frac{2D'}{D}x^2y^2-\frac{D'^2}{D^2}x^2y^2\right]$$

$$= e^{x+y}\left[\frac{1}{D^2}(x^2y^2)+\frac{2}{D^2}\left(2\frac{x^3}{3}y\right)-\frac{1}{D^2}\left(\cancel{2}\frac{x^4}{\cancel{12}_6}\right)\right]$$

$$= e^{x+y}\left[\frac{x^4}{12}y^2+\frac{\cancel{4}}{3}y\frac{x^5}{\cancel{20}_5}-\frac{1}{6}\frac{x^6}{30}\right]$$

$$= e^{x+y}\left[\frac{x^4y^2}{12}+\frac{x^5y}{15}-\frac{x^6}{180}\right]$$

$\therefore$ Solution is $z = C.F + P.I$

$$= f_1(y+x)+x f_2(y+x)+e^{x+y}\left[\frac{x^4y^2}{12}+\frac{x^5y}{15}-\frac{x^6}{180}\right]$$