19MEB201 - Fluid Mechanics and Machinery
UNIT -2 FLOW THROUGH CIRCULAR CONDUITS
NOTES

FLOW THROUGH FLAT PLATE AND CIRCULAR CONDUITS

VISCOUS FLOW – Introduction:

The chapter deals with the flow of fluids which are viscous and flowing at very low velocity. At low velocity the fluid moves in layers. Each layer of fluid slides over the adjacent layer.

Due to relative velocity between two layers the velocity gradient $\frac{du}{dy}$ exists and hence a shear stress $\tau = \mu \frac{du}{dy}$ acts on the layers.

The following cases will be considered in this chapter

1. Flow of viscous fluid through circular pipe
2. Flow of viscous fluid between two parallel plates
3. Kinetic energy correction and momentum correction factors
4. Power absorbed in viscous flow through (a) Journal bearings (b) Foot-step bearing (c) Collar bearings

FLOW OF VISCOUS FLUID THROUGH CIRCULAR PIPE

For the Flow of viscous fluid through circular pipe, the velocity to average velocity, the shear stress distribution and drop of Pressure for a given length to be determined.

The flow through the circular pipe will be viscous or Laminar, if the Reynolds number (Re^*) is Less than 2000. The expression for Reynold number is given by

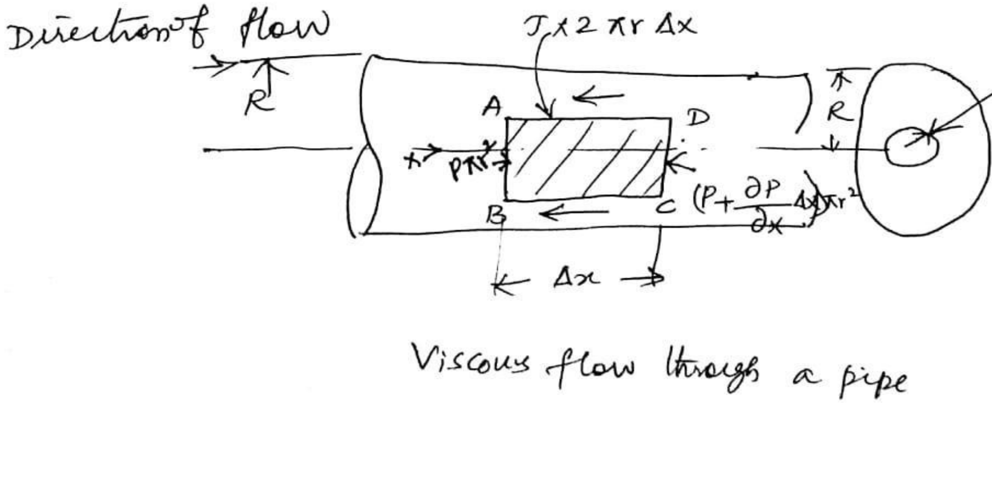
$$Re = \frac{\rho V D}{\mu}$$

ρ – Density of fluid flowing through pipe

V – Average velocity of fluid

D – Diameter of pipe

μ – viscosity of fluid



Viscous flow through a pipe

Consider a horizontal pipe of radius R. The viscous fluid is flowing from left to right in the pipe as shown in figure

(a) consider a fluid element of radius r sliding in a cylindrical fluid element of radius $(r+dr)$ Let the Length of fluid element be Δx If P is the intensity of Pressure on the face AB.

Then the intensity of Pressure on face CD will be $\left(P+\frac{\partial P}{\partial x}\Delta x\right)$. Then the forces acting on the fluid element are

1. The Pressure force $P \times \pi r^2$ on face AB
2. The Pressure force $\left(P+\frac{\partial P}{\partial x}\Delta x\right)\pi r^2$ on face CD
3. The shear force $\tau \times 2\pi r \Delta x$ on the Surface of fluid element. As there is no acceleration hence the Summation of all forces in the direction of flow must be Zero

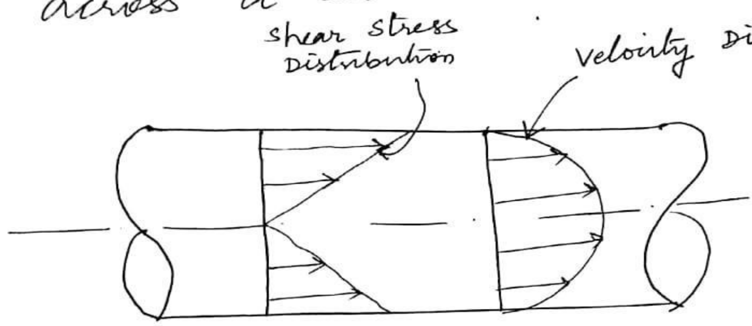
$$P\pi r^2 - \left(P+\frac{\partial P}{\partial x}\Delta x\right)\pi r^2 - \tau \times 2\pi r \times \Delta x = 0$$

$$-\frac{\partial P}{\partial x}\cancel{\Delta x}\,\cancel{\pi r^2} - \tau \times 2\,\cancel{\pi r} \times \cancel{\Delta x} = 0$$

$$\frac{-\partial p}{\partial x} \cdot r - 2\tau = 0$$

$$\boxed{\tau = -\frac{\partial p}{\partial x}\frac{r}{2}} \quad \text{--- (1)}$$

The shear stress τ across a section varies with 'r' as $\frac{\partial p}{\partial x}$ across a section is constant. Hence shear stress distribution across a section is linear.



(a) (b)

Shear stress and velocity distribution across a section

(i) <u>Velocity distribution</u>: To obtain the velocity distribution across a section, the value of shear stress $\tau = \mu \frac{du}{dy}$ is substituted in Eqn — (1)

but in the relation $\tau = \mu \frac{du}{dy}$ y is measured from the pipe wall. Hence

$$y = R - r \text{ and } dy = -dr$$

$$\tau = \mu \frac{du}{-dr} = -\mu \frac{du}{dr}$$

Substituting the value in Eqn ① we get

$$-\mu \frac{du}{dr} = \frac{-\partial P \cdot r}{\partial x \cdot 2}$$

(or)

$$\frac{du}{dr} = \frac{1}{2\mu} \frac{\partial P}{\partial x} \cdot r$$

Integrating the above equation w.r.t r we get

$$\boxed{u = \frac{1}{4\mu} \frac{\partial P}{\partial x} r^2 + c} \quad \text{——— ②}$$

where C is the constant of integration and its value is obtained from the boundary condition that at $r = R$ $u = 0$ iteration

$$0 = \frac{1}{4\mu} \frac{\partial P}{\partial x} R^2 + c$$

$$\boxed{c = -\frac{1}{4\mu} \frac{\partial P}{\partial x} R^2}$$

Substituting the value of c in Eqn ② we get

$$u = \frac{1}{4\mu} \cdot \frac{\partial P}{\partial x} r^2 - \frac{1}{4\mu} \cdot \frac{\partial P}{\partial x} R^2$$

$$\boxed{u = -\frac{1}{4\mu} \frac{\partial P}{\partial x} [R^2 - r^2]} \quad \text{——— ③}$$

In equation ③ we get value of $\mu = \frac{\partial P}{\partial x}$ and R are constant, which means the velocity u varies with the square of r. Thus equation ③ is a equation of Parabola

This shows that the velocity distribution across the section of a pipe is Parabolic. (parallel plate)

(ii) Ratio of maximum velocity to Average velocity

The velocity is maximum, when $r=0$ in equation ③. Thus maximum velocity U_{max} is obtained as

$$U_{max} = -\frac{1}{4\mu}\frac{\partial p}{\partial x}R^2 \quad \text{—— (4)}$$

The average velocity u is obtained by dividing the discharge of the fluid across the section by the area of the pipe (πR^2). The Discharge Q across the section is obtained by considering the flow through a circular ring element of radius r and thickness dr as shown in fig 6.

The fluid flowing per second through this elementary ring

$$dQ = \text{Velocity at a radius } r \times \text{Area of ring element}$$

$$= u \times 2\pi r\, dr.$$

$$= -\frac{1}{4\mu}\frac{\partial p}{\partial x}(R^2-r^2) \times 2\pi r\, dr$$

$$Q = \int_0^R dQ$$

$$= \int_0^R -\frac{1}{4\mu}\frac{\partial p}{\partial x}(R^2-r^2) \times 2\pi r\, dr$$

$$= \frac{1}{4\mu}\left(-\frac{\partial p}{\partial x}\right)2\pi\int_0^R (R^2-r^2)\, r\, dr.$$

$$Q = \frac{1}{4\mu}\left(-\frac{\partial p}{\partial x}\right) 2\pi \int_0^R (R^2 - r^2)\, r \cdot dr.$$

$$= \frac{1}{4\mu}\left(\frac{-\partial p}{\partial x}\right) 2\pi \int_0^R (R^2 r - r^3)\, dr.$$

$$= \frac{1}{4\mu}\left(\frac{-\partial p}{\partial x}\right) 2\pi \left[\frac{R^2 r^2}{2} - \frac{r^4}{4}\right]_0^R$$

$$= \frac{1}{4\mu}\left(-\frac{\partial p}{\partial x}\right) \times 2\pi \left[\frac{R^4}{2} - \frac{R^4}{4}\right]$$

$$= \frac{1}{4\mu}\left(-\frac{\partial p}{\partial x}\right) \cancel{2}\pi \frac{R^4}{\cancel{4}_2}$$

$$Q = \frac{\pi}{8\mu}\left(-\frac{\partial p}{\partial x}\right) R^4$$

Average velocity $\bar{u} = \dfrac{Q}{Area} = \dfrac{\frac{\cancel{\pi}}{8\mu}\left(-\frac{\partial p}{\partial x}\right) R^{\cancel{4}2}}{\cancel{\pi} R^{\cancel{2}}}$

$$\boxed{\bar{u} = \frac{1}{8\mu}\left(\frac{-\partial p}{\partial x}\right) R^2 \quad \text{--- } (5)}$$

Dividing eqn (4) by (5)

$$\frac{U_{max}}{\bar{u}} = \frac{-\frac{1}{4\mu}\frac{\partial p}{\partial x} R^2}{\frac{1}{8\mu}\left(-\frac{\partial p}{\partial x}\right) R^2} = \frac{8}{4} = 2.$$

$\therefore$ Ratio of maximum velocity to Average velocity $= 2$.

$$\boxed{\frac{U_{max}}{\bar{U}} = 2}$$

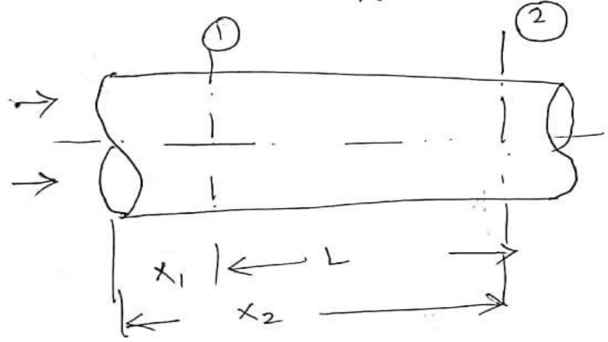
(iii) Drop of Pressure for a given Length (L) of a pipe

from Eqn (5) we have

$$\bar{u} = \frac{1}{8\mu}\left(-\frac{\partial p}{\partial x}\right)R^2$$

(or)

$$-\frac{\partial p}{\partial x} = \frac{8\mu\bar{u}}{R^2}$$



Integrating the above equation w.r.t x we get

$$-\int_2^1 dp = \int_2^1 \frac{8\mu\bar{u}}{R^2}\,dx$$

$$-[P_1 - P_2] = \frac{8\mu\bar{u}}{R^2}[x_1 - x_2]$$

(or)

$$(P_1 - P_2) = \frac{8\mu\bar{u}}{R^2}(x_2 - x_1)$$

$$= \frac{8\mu\bar{u}}{R^2}L \quad \Big|\; x_2 - x_1 = L \text{ from } (3)$$

$$= \frac{8\mu\bar{u}L}{\left(\frac{D}{2}\right)^2}$$

$$\boxed{P_1 - P_2 = \frac{32 \mu \bar{u} L}{D^2}}$$

where $P_1 - P_2$ is the drop of Pressure

$\therefore$ Loss of Pressure head $= \frac{P_1 - P_2}{\rho g}$

$$\boxed{\frac{P_1 - P_2}{\rho g} = h_f = \frac{32 \mu \bar{u} L}{\rho g D^2}} \quad \text{--- (6)}$$

Equation (6) is called Hagen poiseuille formula

(*) HAGEN POISEULLE EQN.