DEPARTMENT OF AGRICULTURAL ENGINEERING

COURSE CODE & NAME: 19MEB201 & FLUID MECHANICS AND MACHINERY

II YEAR / III SEMESTER

UNIT - 3

TOPIC : Need for dimensional analysis –dimensional analysis by using Buckingham’s π theorem method- Similitude –types of similitude

CONTENT

- Dimensional analysis, dimensional homogenty,
- Use of Buckingham - pie theorem,
- Dimensionless numbers,
- Similarity laws,
- Model investigations both submerged and partially submerged bodies.

Dimensional Analysis

The variables involved in physical phenomena are known, while the relationship among the variables is not known. Such a relationship can be formulated between a set of dimensionless groups of variables and the groups numbering less than the variables. This procedure is called *dimensional analysis.*

Uses of Dimensional Analysis:

- To test the dimensional homogeneity of any equation of fluid motion.
- To derive rational formulae for flow phenomenon.
- To drive equation in terms of non dimensional parameters.
- To plan model tests and present experimental results in systematic manner.

Dimensional homogeneity.

A physical equation is the relation between two or more physical quantities, any physical relationship between quantities must be dimensionally homogeneous and numerically homogeneous.

$$\rho = wh$$

$Dimensions\ of\ \rho = ML^{-1}T^{-2}$

$Dimensions\ of\ wh = ML^{-2}T^{-2} \times L = ML^{-1}T^{-2}$

$\therefore \rho = wh$ is **Dimensionally Homogeneous**

Application:

- To determine the dimension of a physical quantities.
- To check whether an equation of any physical quantity is dimensionally homogeneous or not.

Methods of Dimensional analysis:

- Rayleigh's methods
- Buckingham's Pi method.

Rayleigh's methods:

In this method used for determining the expression for a variables which depends upon maximum three or four variables only. Independent variables becomes more than four variables, its is very difficult to find the expression for the dependent variables.

In this method a functional relationship of some variables is expressed in the form of an exponential equation must be dimensionally homogeneous. If X is a dependent variables which depends on $X_1, X_2, X_3 \ldots\ldots X_n$

$$X = f(X_1, X_2, X_3 \ldots\ldots X_n)$$

$$X = C\left(X_1^a, X_2^b, X_3^c \ldots\ldots X_n^n\right)$$

Where C is constant, $a, b, c \ldots n$ are arbitrary powers.

Buckingham's Pi method:

- It states that if an equation involving (k)variables is dimensionally homogeneous, it can be reduced to a relationship among $(k-r)$independent dimensionless products, where $r$is the minimum number of reference dimensions required to describe the variable.

Mathematically, $y = f(x_1, x_2 \ldots \ldots .., x_k)$

The dimensions of the variables on the left side of the equation are equal to the dimensions of any term on the right side of equation, then, it is possible to rearrange the above equation into a set of dimensionless products (*pi terms*)

$$\pi_1 = \emptyset(\pi_2, \pi_3 \ldots \ldots ., \pi_{k-r})$$

where $\emptyset(\pi_2, \pi_3 \ldots \ldots ., \pi_{k-r})$is a function of $\pi_2$through π_{k-r}.

- The required number of *pi terms* is less than the number of original variables by(r), where $(r)$is determined by the minimum number of reference dimensions required to describe the original list of variables. These reference dimensions are usually the basic dimensions $M, L\ and\ T$(Mass, Length and Time).

Step I: List out all the variables that are involved in the problem.

Geometry Property: Diameter, Length, etc.

Fluid Property: Density, Viscosity, etc.

Flow Property: Force, Pressure, Velocity, etc.)

Step II: Express each variable in terms of basic dimensions.

The basic dimensions will be either $M, L\ and\ T$ or $F, L\ and\ T.$

Step III: Decide the required number of pi terms.

Step IV: Select the number of repeating variables.

Step V: Formation of pi terms:

Step VI: Checking of pi terms:

Step VII: Final form of relationship among *pi terms:*

Problem:2

A thin rectangular plate having width (w) and height (h) is located so that it is normal to a moving stream of fluid. Assume the drag (F) that the fluid exerts on the plate is a function of w and h, the fluid viscosity and density μ and ρ, respectively and the velocity V of the fluid approaching the plate. Determine a suitable set of pi terms to study this problem experimentally.

Solution:

- Given, for a smooth pipe, $F = f(w, h, \mu, \rho, V)$
- The dimensional parameters involved are F, w, h, μ, ρ, V i.e. $k = 6$
- Primary / Reference dimensions are M, L, T i.e. $r = 3$
- Number of *pi terms* required $= k - r = 3$
- Number of repeating parameters $= m = r = 3$
- Dimensions of each parameter in terms of reference dimension are

$$F = MLT^{-2},\ \ \rho = ML^{-3}, \mu = ML^{-1}T^{-1}, V = LT^{-1}, w = L, \text{ and } h = L$$

First *pi term* is written as

$$\pi_1 = Fw^aV^b\rho^c$$

$$M^0L^0T^0 = (MLT^{-2})(L)^a(LT^{-1})^b(ML^{-3})^c$$

Equating the exponents of M, L, T ,

$$M: 1 + c = 0 \qquad \Rightarrow c = -1$$

$$L: 1 + a + b - 3c = 0 \qquad \Rightarrow a = -2$$

$$T: -2 - b = 0 \qquad \Rightarrow b = -2$$

$$\boldsymbol{\pi_1 = \frac{F}{w^2V^2\rho}}$$

The same procedure is repeated with the other non-repeating variable h to find the second *pi term* i.e.

$$\pi_2 = hw^dV^e\rho^f$$

$$M^0L^0T^0 = (L)(L)^d(LT^{-1})^e(ML^{-3})^f$$

Equating the exponents of M, L, T

$$M: f = 0$$

$$L: 1 + d + e - 3f = 0 \quad \Rightarrow d = -1$$

$$T: e = 0$$

$$\boldsymbol{\pi_2 = \frac{h}{w}}$$

The third *pi term* is formed with the remaining non-repeating variable μ so that

$$\pi_3 = \mu w^g V^h \rho^i$$

$$M^0 L^0 T^0 = ML^{-1}T^{-1}(L)^g(LT^{-1})^h(ML^{-3})^i$$

Equating the exponents of M, L, T

$$M: 1 + i = 0 \qquad \Rightarrow i = -1$$

$$L: -1 + g + h - 3i = 0 \quad \Rightarrow g = -1$$

$$T: -1 - h = 0 \qquad \Rightarrow h = -1$$

$$\boldsymbol{\pi_3 = \frac{\mu}{wV\rho}}$$

Now, checking the dimensionality of *pi term*, we have

$$\pi_1 = \frac{F}{w^2V^2\rho} = \frac{(F)}{(L)^2(LT^{-1})^2(FL^{-4}T^2)} = F^0L^0T^0$$

$$\pi_2 = \frac{h}{w} = \frac{L}{L} = F^0L^0T^0$$

$$\pi_3 = \frac{\mu}{DV\rho} = \frac{(FL^{-2}T)}{(L)(FL^{-4})(LT^{-1})} = F^0L^0T^0$$

The functional relationship can be written as,

$$\boldsymbol{\frac{F}{w^2V^2\rho} = \emptyset\left(\frac{h}{w}, \frac{\mu}{DV\rho}\right)}$$

$$\boldsymbol{F = w^2V^2\rho\emptyset\left(\frac{h}{w}, \frac{1}{R_e}\right)}$$

Where R_e is the Reynolds number

Types of forces:

Inertia force

It is defines as the product of mass and acceleration of the flowing fluid and acts in the direction opposite to the direction of acceleration.

Viscous force:

It is defines as the product of shear stress due to viscosity and surface area of the flow.

Gravity force:

It is defines as the product of mass and acceleration due to gravity.

Pressure force:

It is defines as the product of pressure intensity and cross-sectional area of the flowing fluid.

Surface tension force:

It is defines as the product of surface tension and length of surface of the flowing fluid.

Elastic force:

Product of elastic stress and area of the flowing fluid.

DIMENSIONALESS NUMBERS

Reynolds number(R_e)*:*

It is defined as the ratio of inertia force to viscous force.

Inertia Force $= mass \times Acceleration$

Viscous Force $= Shear\ stress \times Area$

$$R_e = \frac{\rho V L}{\mu} = \frac{V L}{\mathrm{U}}$$

Examples:

- Motion of submarine completely under water.
- Low velocity motion around automobiles and aeroplanes.
- Incompressible flow through pipes of smaller sizes.
- Flow through low speed in turbo machine.

Euler number E_u:

It is defined as the ratio of inertia force to pressure force.

pressure force $= Intensity\ of\ pressure \times Area$

$$E_u = \frac{V}{\sqrt{P/\rho}}$$

Examples:

- Discharge throw orifices, mouth pieces and sluices.
- Pressure rise due to sudden closure of valves.
- Flow through pipes.
- Water hammer created in penstocks.

Froude number(F_r):

It is interpreted as the ratio of inertia force to gravity force.

Gravity Force= $mass \times acceleration\ due\ to\ gravity$

$$F_r = \frac{V}{\sqrt{g.L}}$$

Example:

- Flow over a notches and weirs.
- Flow over the spillway of a dam.
- Flow through open channels, waves and jumps.
- Motion of ship in rough and turbulent sea.

Weber number(W_e):

It is defined as the ratio of the inertia force to surface tension force.

surface tension force= $surface\ tension\ \times length$

$$W_e = \frac{\rho V^2 L}{\sigma}$$

Examples:

- Capillary movement of water in soil.
- Flow of blood in veins and arteries, Liquid atomisation.

Mach number (M_a):

It is defined as the ratio of inertia force to compressibility force.

$$M_a = \frac{V}{c} = \frac{V}{\sqrt{\frac{dp}{d\rho}}} = \frac{V}{\sqrt{\frac{E_v}{\rho}}}$$

Examples: high velocity of flow in pipes, motion of high speed projectiles.

Model analysis:

Model:

Model is the small scale replica of the actual structure of the machine.

Prototype:

Actual structure or machine is called prototype.

- A model is smaller than the prototype so as to conduct laboratory studies and it is less expensive to construct and operate.
- Models are larger than the prototype

E.g. study of the motion of blood cells whose sizes are of the order of micrometres.

Advantages:

- Model test are quite economical and convenient (because design, construction and operation of model can be change).
- The use of models the performance hydraulic structures can be predicated in advance.
- It can be used to detect and rectify the defects of an existing structure which not functioning properly.

Applications of the model testing:

- Civil engineering structures Eg. Dams, spillways, weirs, cannels etc.
- Flood control, investigation of silting, scour in rivers and irrigation channels.
- Turbine, pumps and compressors.
- Design of harbours, ships and submarines.
- Aeroplane, rockets and missiles.
- Tall building.

Similitude:

Similitude is the indication of a known relationship between a model and prototype.

Geometric similarity:

A model and prototype are geometric similar. Ratio of corresponding length in the model and in the prototype must be the same.

Kinematic similarity :

The motions of two systems are kinematically similar if homogeneous particles lie at homogeneous points at homogeneous times.

Dynamic similarity :

When two flows have force distributions such that identical types of forces are parallel and are related in magnitude by a constant scale factor at all corresponding points, then the flows are dynamic similar.

Model law:

Dynamic similarity between model and prototype be made equal.

That means dimensionless number should be same for the model as well as prototype. The models are design on the basics of the force which is dominating in the flow situation. The laws on which models are design for dynamic similarity are called as model or similarity law.

Reynolds model laws:

The flow situations to inertia, viscous force is the only other predominates force.

Applications:

- Motion of airplanes
- Flow of incomprssive fluid in a closed pipes.
- motion submarines completely under water.
- Flow around structures and other bodies immersed completely under fluids.

Froude model law:

when the gravitational force can consider to be the only predominant force which controls the motion in addition to the inertia force.

Applications:

- Free surface flow, flow over a spillways, sluices etc.
- Flow of jet from an orifice or nozzle.
- Fluids of different mass density flow over one another.

Euler's model law:

In a fluid system where pressure forces alone are the controlling forces in addition to the inertia force.

Applications:

Enclosed fluid system where the turbulence fully develop so that viscous forces, gravity and surface tension forces are negligible.

Weber model law:

In a fluid system where surface tension effects predominate in addition to inertia force.

Applications:

- Flow over weirs involving very low head.
- Very thin sheet of liquid flowing over a surface.
- Capillary waves in channels.
- Capillary rise in narrow passages.
- Capillary movement of water in soil.

Mach model law:

When in any fluid system only the forces resulting from elastic compression are significant in addition to inertia force.

Applications:

- Aero dynamic testing
- Hydraulic model testing of unsteady flow.
- Under water testing of torpedoes.

Types of models:

Undistorted model:

It is one which is geometrically similar to its prototype.

The condition of similitude are completely satisfied. the model test are used to predict the performance of prototype body.

Distorted model:

It is one which is not geometrically similar to its prototype.

This model different scale ratio for the linear dimensions are adopted.

Examples:

- Rivers, harbours and estuaries.
- Reason for adopting distorted model.
- Maintaining accuracy in vertical dimension.
- Maintaining turbulent flow.
- Obtaining suitable roughness condition, bed material and adequate movement.

Scale effect in model:

Model testing is not possible to predict the exact behaviour of the prototype. The behaviour of prototype as predicted by two models with different scale ratio. Such difference in the predication of behaviour of the prototype is termed as *scale effect.*

Limitation of hydraulic similitude:

- The model results are qualitative but not quantitate.
- Compared to the cost of analytical work models are usually expensive.
- The selection of size of model is a matter of experience.

THANK YOU