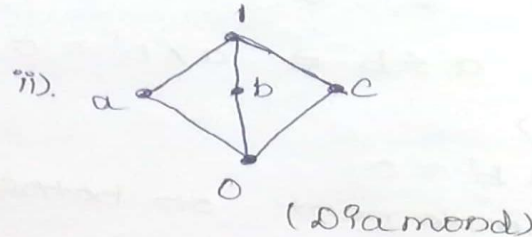
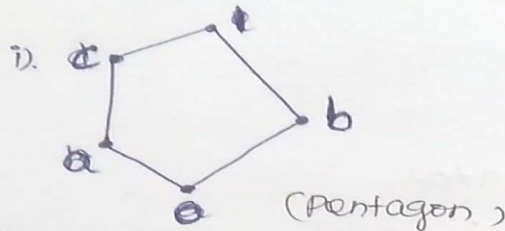




## UNIT 5- LATTICES AND BOOLEAN ALGEBRA

## Direct product and homomorphism

Determine which of the following lattices are modular.



1. Consider  $(a, b, c)$

clearly  $a \leq c$

$$\text{Now LHS} = a \vee (b \wedge c)$$

$$= a \vee \underline{a}$$

$$= a$$

$$\text{RHS} = (a \vee b) \wedge c$$

$$= \underline{c} \wedge c$$

$$= c$$

$$a \neq c$$

If  $a \leq c$ , then  $a \vee (b \wedge c) \neq (a \vee b) \wedge c$

∴ condition is not satisfied.

∴ Pentagon lattice is not a modular lattice.

ii). Diamond lattice is modular.

Theorem:

Every distributive lattice is modular but not conversely.

Proof

Let  $(L, \wedge, \vee)$  be the given distributive lattice.

$$D_1: a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c), \quad \forall a, b, c \in L$$

$$\text{If } a \leq c \text{ then } a \vee c = c \xrightarrow{b \cap} c$$

$$(1) \Rightarrow a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$$



# SNS COLLEGE OF TECHNOLOGY

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UNIT 5- LATTICES AND BOOLEAN ALGEBRA

Direct product and homomorphism

$$= (a \vee b) \wedge c \quad \text{using (2)}$$

$\therefore$  If  $a \leq c$  then  $(a \vee b) \wedge c = a \vee (b \wedge c)$

$\therefore$  Every distributive lattice is modular.

But, converse is not true.  $\checkmark$

i.e., Every modular lattice need not be distributive.

For eg.,  $M_5$  (Diamond lattice) is modular but it is not distributive.