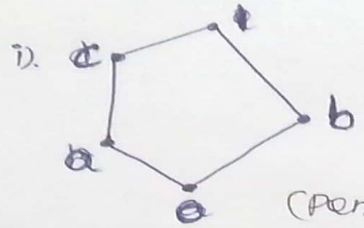
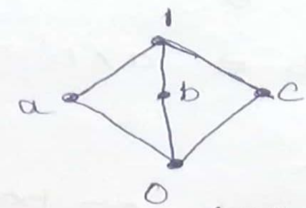
Determine which of the following lattices are modular.

i). 

(Pentagon)

ii). 

(Diamond)

i). consider (a, b, c)

clearly $a \le c$

Now $LHS = a \vee (b \wedge c)$

$= a \vee \underline{a}$

$= a$

$RHS = (a \vee b) \wedge c$

$= \underline{c} \wedge c$

$= c$

$a \ne c$

If $a \le c$, then $a \vee (b \wedge c) \ne (a \vee b) \wedge c$

M, condition is not satisfied.

N_5) Pentagon lattice is not a modular lattice.

ii). Diamond lattice is modular.

Theorem:

Every distributive lattice is modular but not conversely.

Proof.

Let $(L, \wedge, \vee)$ be the given distributive lattice.

D_1: $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$, $\forall a, b, c \in L$ $\to (1)$

If $a \le c$ then $a \vee c = c \longrightarrow (2)$

$(1) \Rightarrow$ $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$

$= (a \vee b) \wedge c$ using (2)

$\therefore$ If $a \leq c$ then $(a \vee b) \wedge c = a \vee (b \wedge c)$

$\therefore$ Every distributive lattice is modular.

But, converse is not true.

i.e., Every modular lattice need not be distributive.

For eg., M_5 (Diamond lattice) is modular but it is not distributive.