Fourier Cosine Transform :

The infinite Fourier Cosine Transform of $f(x)$ is defined by

$$F_c[f(x)] = \sqrt{\frac{2}{\pi}} \int_0^\infty f(x) \cos(sx)\, dx = F_c(s)$$

The inverse Fourier Cosine transform denoted by $F_c^{-1}[F_c[f(x)]]$ is defined by

$$f(x) = \sqrt{\frac{2}{\pi}} \int_0^\infty F_c[f(x)] \cos sx \, ds .$$

$F_c[f(x)]$ and $F_c^{-1}[F_c[f(x)]]$ are called Fourier Cosine Transform Pair.

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Problem 1:

Find the Fourier Cosine Transform of $f(x)$

$$f(x) = \begin{cases} 1 , & 0 < x < a \\ 0 , & x > a \end{cases}$$

Solution: Fourier Cosine Transform

$$F_c[f(x)] = \sqrt{\frac{2}{\pi}} \int_0^\infty f(x) \cos(sx)\, dx .$$

$$F_c[1] = \sqrt{\frac{2}{\pi}} \int_0^a 1 \cos(sx)\, dx$$

$$= \sqrt{\frac{2}{\pi}} \left[\frac{\sin(sx)}{s}\right]_0^a = \sqrt{\frac{2}{\pi}} \left[\frac{\sin as}{s} - \frac{\sin 0}{s}\right]$$

$$= \sqrt{\frac{2}{\pi}} \frac{1}{s} \left[\sin as - 0\right] = \sqrt{\frac{2}{\pi}} \frac{\sin(as)}{s}$$

2). Find the Fourier Cosine Transform of e^{-ax} ~~sin ax~~, $a > 0$

$$F_c[f(x)] = \sqrt{\frac{2}{\pi}} \int_0^\infty f(x) \cos(sx)\, dx$$

$$= \sqrt{\frac{2}{\pi}} \int_0^\infty e^{-ax} \cos(sx)\, dx$$

$$= \sqrt{\frac{2}{\pi}} \left(\frac{a}{s^2+a^2}\right)$$

formula

$$\int_0^\infty e^{-ax} \cos bx \, dx = \frac{a}{b^2+a^2}$$

$\int_0^\infty e^{-ax} \sin bx\, dx$

(3)

5) Find the Fourier Cosine Tranform of $5e^{-2x}+2e^{-5x}$.

Soln:

$$F_c[f(x)] = \sqrt{\frac{2}{\pi}}\int_0^\infty f(x)\cos(sx)\,dx$$

$$F_c[5e^{-2x}+2e^{-5x}] = \sqrt{\frac{2}{\pi}}\int_0^\infty (5e^{-2x}+2e^{-5x})\cos sx\,dx$$

$$= \sqrt{\frac{2}{\pi}}\left\{5\int_0^\infty e^{-2x}\cos(sx)\,dx + 2\int_0^\infty e^{-5x}\cos(sx)\,dx\right.$$

$$= \sqrt{\frac{2}{\pi}}\left\{5\,\frac{2}{s^2+4} + 2\cdot\frac{5}{s^2+25}\right\}$$

$$\int_0^\infty e^{-ax}\cos bx\,dx = \frac{a}{a^2+b^2}$$

$$= 10\sqrt{\frac{2}{\pi}}\left[\frac{1}{s^2+4} + \frac{1}{s^2+25}\right]$$

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HW Find the cosine Transform of $f(x)$

$$f(x) = \begin{cases} x & 0 < x < 1 \\ 2-x & 1 < x < 2 \\ 0 & x > 2 \end{cases}$$

$\sqrt{\frac{2}{\pi}} \cdot \frac{2\cos s}{s^2}(1-\cos s)$

formula: $\cos^2 x = \frac{1+\cos 2x}{2}$

$F_c[f(x)]$

$$= \sqrt{\frac{2}{\pi}}\left[\frac{\sin s}{s} + \frac{\cos s}{s^2} - \frac{1}{s^2} - \frac{\cos 2s}{s^2} - \frac{\sin s}{s} + \frac{\cos s}{s^2}\right]$$

$$= \sqrt{\frac{2}{\pi}}\left[\frac{2\cos s}{s^2} - \frac{1}{s^2} - \frac{\cos 2s}{s^2}\right]$$

$$= \sqrt{\frac{2}{\pi}} \cdot \frac{1}{s^2}\left[2\cos s - 1 - \cos 2s\right]$$

$$= \sqrt{\frac{2}{\pi}} \cdot \frac{1}{s^2}\left[2\cos s - (1 + \cos 2s)\right]$$

$$= \sqrt{\frac{2}{\pi}} \cdot \frac{1}{s^2}\left[2\cos s - 2\cos^2 s\right]$$

$$= \sqrt{\frac{2}{\pi}} \cdot \frac{1}{s^2}\, 2\cos s\left[1 - \cos s\right]$$

$$F_c[f(x)] = \sqrt{\frac{2}{\pi}}\, \frac{2\cos s}{s^2}(1-\cos s)$$