Unit-2. Fourier Transform

1) Find the Fourier Transform of $f(x) = \begin{cases} x & \text{if } |x| \le a \\ 0 & \text{if } |x| > a \end{cases}$

2) Find the Fourier transform of $f(x) = \begin{cases} 1 & \text{if } |x| \le a \\ 0 & \text{if } |x| > a \end{cases}$
& deduce that $\int_0^\infty \frac{\sin t}{t}\,dt$ & $\int_0^\infty \left(\frac{\sin t}{t}\right)^2 dt$

3) Find the Fourier transform of the function $f(x) = \begin{cases} a^2 - x^2, & |x| \le a \\ 0, & |x| > a \end{cases}$
& deduce that $\int_0^\infty \frac{\sin s - s\cos s}{s^3}\,ds$ & $\int_0^\infty \left(\frac{\sin s - s\cos s}{s^3}\right)^3 ds$

4) Find the Fourier transform of the function $f(x) = \begin{cases} 1-x^2, & |x| \le 1 \\ 0, & |x| > 1 \end{cases}$
& deduce that $\int_0^\infty \left(\frac{\sin t - t\cos t}{t^3}\right) dt$ & $\int_0^\infty \left(\frac{\sin t - t\cos t}{t^3}\right)^2 dt$

5) Find the Fourier transform of the function $f(x) = \begin{cases} 1-|x|, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$
& deduce that $\int_0^\infty \left(\frac{\sin t}{t}\right)^2 dt$ & $\int_0^\infty \left(\frac{\sin t}{t}\right)^4 dt$

6) Find the Fourier transform of the function $f(x) = \begin{cases} a-|x|, & |x| \le a \\ 0, & |x| > a \end{cases}$
& deduce that $\int_0^\infty \left(\frac{\sin t}{t}\right)^2 dt$ & $\int_0^\infty \left(\frac{\sin t}{t}\right)^4 dt$.

7) Find the Fourier transform of $f(x) = e^{-a^2x^2}$

8) Show that the function $e^{-x^2/2}$ is self reciprocal under FT.

9) Find Fourier sine & cosine transform of e^{-ax}

10) Find the sine & cosine transform of the function $f(x) = \frac{e^{-ax}}{x}$

11) Find the sine & cosine transform of xe^{-ax}

12) Evaluate using FT

(i) $\int_0^\infty \frac{dx}{(x^2+a^2)(x^2+b^2)}$ (ii) $\int_0^\infty \frac{x^2\,dx}{(x^2+a^2)(x^2+b^2)}$