SNS COLLEGE OF TECHNOLOGY

Internal Assessment-1 (SET-A)

Academic Year 2023-2024 (ODD)

19MAT201 TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

PART-A

(1) A function $f(x)$ can be expanded as an infinite trigonometric series if it satisfies the following:

(i) $f(x)$ is periodic, single valued and finite.

(ii) $f(x)$ has finite number of finite discontinuities.

(iii) $f(x)$ has finite number of maxima or minima.

(2) $$f(x) = \sum_{n=1}^{\infty} \frac{2}{n\pi}[1-(-1)^n] \sin nx$$

(3) $$\text{RMS Value} = \sqrt{\frac{\int_a^b [f(x)]^2 dx}{b-a}}$$

(4) $$F[f(x)] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{isx} dx$$

$$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(s) e^{-isx} dx$$

(5) $$F[f(x)] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{isx} dx$$

Put $ax = y$

$a\,dx = dy$

$dx = \frac{dy}{a}$

$$F[f(ax)] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(ax) e^{isx} dx$$

$$= \frac{1}{a} F(s/a)$$

Part-B

(6) a) (i) $a_0 = \frac{4l^2}{3}$, $a_n = -\frac{4}{n^2}$, $b_n = 0$

$$f(x) = \frac{2l^2}{3} - \sum_{n=1}^{\infty} \frac{4}{n^2} \cos\left(\frac{n\pi x}{l}\right)$$

(ii) $b_n = \frac{(-1)^n}{n}$ & $f(x) = \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \sin nx$

(b) $a_0 = \frac{2\pi^2}{3}$, $a_n = \frac{4}{n^2}(-1)^n$, $b_n = 0$

$$f(x) = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \cos nx$$

Put $x = \pi \Rightarrow \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots \infty = \frac{\pi^2}{6}$

Apply parseval's identity, $\frac{2}{\pi}\int_0^{\pi} [f(x)]^2 dx = \frac{a_0^2}{2} + \sum_{n=1}^{\infty} a_n^2$

$\Rightarrow \frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \cdots \infty = \frac{\pi^4}{90}$.

(7) a) $F(s) = \frac{2}{\sqrt{2\pi}}\left[\frac{1 - \cos as}{s^2}\right]$

Using Fourier inversion formula & substituting $x = 0, a = 2,$ $s = t$, we get $\int_0^{\infty} \left(\frac{\sin t}{t}\right)^2 dt = \pi/2$ & applying Parseval identity we get $\int_0^{\infty} \left(\frac{\sin t}{t}\right)^4 dt = \pi/3$

b) (i) $a_0 = 4/3$, $a_n = \frac{-4}{n^2\pi^2}$, $b_n = 0$

$$f(x) = \frac{2}{3} - \frac{4}{\pi^2}\sum_{n=1}^{\infty} \frac{\cos n\pi x}{n^2}$$

(ii) $a_0 = l$, $a_n = \frac{2l}{n^2\pi^2}[1 - (-1)^n]$

(8) (a) $F[f(x)] = \sqrt{\frac{2}{\pi}} \cdot \frac{\sin sa}{s}$

Apply Fourier inverse transform & deduce $\int_0^\infty \frac{\sin t}{t} dt = \pi/2$

Using Parseval identity $\int_{-\infty}^{\infty} [F(s)]^2 ds = \int_{-\infty}^{\infty} [f(x)]^2 dx$

we get $\int_0^\infty \left(\frac{\sin t}{t}\right)^2 dt = \pi/2$

b) $a_0 = \frac{2\Sigma y}{N} = \frac{2 \times 4.5}{6} = 1.5$

$a_1 = \frac{2\Sigma y \cos x}{N} = \frac{2 \times 1.12}{6} = 0.37$

$a_2 = \frac{2\Sigma y \cos 2x}{N} = \frac{2 \times 2.668}{6} = 0.889$

$b_1 = \frac{2\Sigma y \sin x}{N} = \frac{2 \times 3.013}{6} = 1.004$

$b_2 = \frac{2\Sigma y \sin 2x}{N} = \frac{2 \times (-0.334)}{6} = -0.11$

$f(x) = \frac{a_0}{2} + \cancel{\Sigma}\ a_1 \cos x + a_2 \cos 2x + b_1 \sin x + b_2 \sin 2x$

$f(x) = 0.75 + 0.37 \cos x + 0.889 \cos 2x + 1.004 \sin x - 0.11 \sin 2x$