EIGEN VALUES AND EIGEN VECTORS

EIGEN VALUES:

The values of λ obtained from the Characteristic equation $|A - \lambda I| = 0$are called Eigen values of matrix A

EIGEN VECTORS:

Let A be a square matrix of order 3 and λ be a scalar(Eigen value). The column matrix

$X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$which satisfies $[A - \lambda I]X = 0$ is called Eigen vector

SYMMETRIC MATRIX

A square matrix $A = [a_{ij}]$ is said to be symmetric if $A^T = A$

[i.e, If the transpose of a matrix is equal to the matrix itself]

Thus, for a symmetric matrix $A = [a_{ij}]$, $a_{ij} = a_{ji}$ for all i and j

PROBLEMS ON NON SYMMETRIC MATRICES WITH NON REPEATED EIGEN VALUES

1. Find the eigen values and eigen vectors of $\begin{bmatrix} 4 & -20 & -10 \\ -2 & 10 & 4 \\ 6 & -30 & -13 \end{bmatrix}$

Solution:

To find the characteristics Equation and eigen values

Let $A = \begin{bmatrix} 4 & -20 & -10 \\ -2 & 10 & 4 \\ 6 & -30 & -13 \end{bmatrix}$

The Characteristic equation is given by

$$\lambda^3 - c_1\lambda^2 + c_2\lambda - c_3 = 0$$

Where

c_1= Sum of leading diagonal elements

$$= 4 + 10 - 13$$

$$= 1$$

c_2=Sum of the minors of leading diagonal elements.

$$= \begin{vmatrix} 10 & 4 \\ -30 & -13 \end{vmatrix} + \begin{vmatrix} 4 & -10 \\ 6 & -13 \end{vmatrix} + \begin{vmatrix} 4 & -20 \\ -2 & 10 \end{vmatrix}$$

$$= (-130 + 120) + (-52 + 60) + (40 - 40)$$

$$= -2$$

c_3=det[A]

$$= \begin{vmatrix} 4 & -20 & -10 \\ -2 & 10 & 4 \\ 6 & -30 & -13 \end{vmatrix}$$

$= 4(-130 + 120) + 20(26 - 24) - 10(60 - 60)$

$= -40 + 40 + 0$

$= 0$

Sub the values in characteristic equation

$$\lambda^3 - c_1\lambda^2 + c_2\lambda - c_3 = 0$$

We get $\quad \lambda^3 - \lambda^2 - 2\lambda = 0$

To find eigen values

$\lambda^3 - \lambda^2 - 2\lambda = 0$

$\lambda(\lambda^3 - \lambda^2 - 2) = 0$

$\lambda = 0$, $\quad (\lambda^3 - \lambda^2 - 2) = 0$

$\lambda = 2\,, \qquad \lambda = -1$

The eigen values are 0,2,-1.

To find the eigen vectors

The eigen vector $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ is given by $(A - \lambda I)X = 0$

$$\begin{bmatrix} 4-\lambda & -20 & -10 \\ -2 & 10-\lambda & 4 \\ 6 & -30 & -13-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left.\begin{aligned} (4-\lambda)x_1 + 20x_2 - 10x_3 = 0 \\ -2x_1 + (10-\lambda)x_2 + 4x_3 = 0 \\ 6x_1 - 30x_2 - (13+\lambda)x_3 = 0 \end{aligned}\right\} \qquad (1)$$

Case(i)

When $\lambda = 0$, the system of equations (1) becomes

$$4x_1 - 20x_2 - 10x_3 = 0 \rightarrow \ (2)$$

$$-2x_1 + 10x_2 + 4x_3 = 0 \rightarrow \ (3)$$

$$6x_1 - 30x_2 - 13x_3 = 0 \rightarrow \ (4)$$

Taking equation (2) and (3), Applying cross rule method we get

$x_1 x_2 \qquad x_3$

-20 -10 4 -20

10 4 -2 10

$$\frac{x_1}{-80+100} = \frac{x_2}{20-16} = \frac{x_3}{40-40}$$

$\frac{x_1}{20} = \frac{x_2}{4} = \frac{x_3}{0}$

The eigen vector is $X_1 = \begin{bmatrix} 5 \\ 1 \\ 0 \end{bmatrix}$

Case(ii)

When $\lambda = -1$, the system of equations (1) becomes

$$5x_1 - 20x_2 - 10x_3 = 0 \rightarrow (5)$$

$$-2x_1 + 11x_2 + 4x_3 = 0 \rightarrow (6)$$

$$6x_1 - 30x_2 - 12x_3 = 0 \rightarrow (7)$$

Taking equation (5) and (6), Applying cross rule method we get

x_1	x_2	x_3	
-20	-10	5	-20
11	4	-2	11

$$\frac{x_1}{-80 + 110} = \frac{x_2}{20 - 20} = \frac{x_3}{55 - 40}$$

$\frac{x_1}{30} = \frac{x_2}{0} = \frac{x_3}{15}$

The eigen vector is $X_2 = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$

Case(iii)

When $\lambda = 2$, the system of equations (1) becomes

$$2x_1 - 20x_2 - 10x_3 = 0 \rightarrow (8)$$

$$-2x_1 + 8x_2 + 4x_3 = 0 \rightarrow (9)$$

$$6x_1 - 30x_2 - 15x_3 = 0 \rightarrow \quad (10)$$

Taking equation (8) and (9), Applying cross rule method we get

$x_1 x_2 \qquad x_3$

-20	-10	2	-20
84	-2	8	

$$\frac{x_1}{-80+80} = \frac{x_2}{20-8} = \frac{x_3}{16-40}$$

$\frac{x_1}{0} = \frac{x_2}{12} = \frac{x_3}{-24}$

The eigen vector is $X_3 = \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}$

Characteristic Equation	Eigen Values	Eigen Vectors
$\lambda^3 - \lambda^2 - 2\lambda = 0$	$\lambda_1 = 0$	$X_1 = \begin{bmatrix} 5 \\ 1 \\ 0 \end{bmatrix}$
	$\lambda_2 = -1$	$X_2 = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$
	$\lambda_3 = 2$	$X_3 = \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}$