DEFINITION:

A system of mn numbers arranged in a rectangular array along m rows and n columns is called mxn matrix

$$A = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{bmatrix}$$

Here A is the matrix of order mxn. It has m rows and n columns. Each of 'm''n' numbers is called an element of the matrix. Matrix A is denoted by A=[a_{ij}]

CHARACTERISTIC EQUATION

Let A be a given matrix. Let λ be a scalar. The equation det[A-λI]=0 or |A-λI|=0 is called the characteristic equation of the matrix A.

Problems

1. Find the characteristic equation of $A = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$

Solution:

The Characteristic equation is given by |A-λI|=0

$\begin{vmatrix} 1 & -1 \\ 0 & 1 \end{vmatrix} - \lambda \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}=0$

$\begin{vmatrix} 1 & -1 \\ 0 & 1 \end{vmatrix} - \begin{vmatrix} \lambda & 0 \\ 0 & \lambda \end{vmatrix}=0$

$$\begin{vmatrix} 1-\lambda & -1 \\ 0 & 1-\lambda \end{vmatrix} = 0$$

$(1-\lambda)^2 = 0$

$\lambda^2 - 2\lambda + 1 = 0$is the required Characteristic equation

2. Find the characteristic equation of $A = \begin{bmatrix} 1 & 2 \\ -3 & 4 \end{bmatrix}$

Solution:

The Characteristic equation is given by $|A-\lambda I|=0$

i.e.,

$$\begin{vmatrix} 1 & 2 \\ -3 & 4 \end{vmatrix} - \lambda \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 0$$

$$\begin{vmatrix} 1 & 2 \\ -3 & 4 \end{vmatrix} - \begin{vmatrix} \lambda & 0 \\ 0 & \lambda \end{vmatrix} = 0$$

$$\begin{vmatrix} 1-\lambda & 2 \\ -3 & 4-\lambda \end{vmatrix} = 0$$

$(1-\lambda)(4-\lambda)+6=0$

$4-\ \lambda-4\lambda+\lambda^2\ +6=0$

$\lambda^2-5\lambda+10=0$is the required Characteristic Equation.

(Alter Method)

To find the Characteristic Equation of A (3 x 3 matrix) is

$$\lambda^3-c_1\lambda^2+c_2\lambda-c_3=0$$

Where

c_1= Sum of leading diagonal elements

c_2=Sum of the minors of leading diagonal elements.

c=det[A]

Problems:

1. Find the characteristic equation of $A=\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$

Solution:

The Characteristic equation is given by

$$\lambda^3 - c_1\lambda^2 + c_2\lambda - c_3 = 0$$

Where

c_1= Sum of leading diagonal elements

$$= -2 + 1 + 0$$

$$= -1$$

c_2=Sum of the minors of leading diagonal elements.

$$= \begin{vmatrix} 1 & -6 \\ -2 & 0 \end{vmatrix} + \begin{vmatrix} -2 & -3 \\ -1 & 0 \end{vmatrix} + \begin{vmatrix} -2 & 2 \\ 2 & 1 \end{vmatrix}$$

$$= -12 - 3 - 6$$

$$= -21$$

c_3=det[A]

$$= \begin{vmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{vmatrix}$$

$= -2(-12) - 2(-6) - 3(-3)$

$= 24 + 12 + 9$

$= 45$

Sub the values of a_1, a_2, a_3 in characteristic equation

$$\lambda^3 - c_1\lambda^2 + c_2\lambda - c_3 = 0$$

We get $\lambda^3 + \lambda^2 - 21\lambda - 45 = 0$

2. Find the characteristic equation of $A = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 3 & -1 \\ 1 & -1 & 3 \end{bmatrix}$

Solution:

The Characteristic equation is given by

$$\lambda^3 - c_1\lambda^2 + c_2\lambda - c_3 = 0$$

Where

c_1= Sum of leading diagonal elements

$$= 3 + 3 + 3$$

$$= 9$$

c_2=Sum of the minors of leading diagonal elements.

$$= \begin{vmatrix} 3 & -1 \\ -1 & 3 \end{vmatrix} + \begin{vmatrix} 3 & 1 \\ 1 & 3 \end{vmatrix} + \begin{vmatrix} 3 & 1 \\ 1 & 3 \end{vmatrix}$$

$$= 8 + 8 + 8$$

$$= 24$$

c_3=det[A]

$$= \begin{vmatrix} 3 & 1 & 1 \\ 1 & 3 & -1 \\ 1 & -1 & 3 \end{vmatrix}$$

$$= 3(8) - 1(4) + 1(-4)$$

$$= 24 - 4 - 4$$

$$= 16$$

Sub the values of c_1, c_2, c_3 in characteristic equation $\lambda^3 - c_1\lambda^2 + c_2\lambda - c_3 = 0$

We get $\lambda^3 - 9\lambda^2 + 24\lambda - 16 = 0$