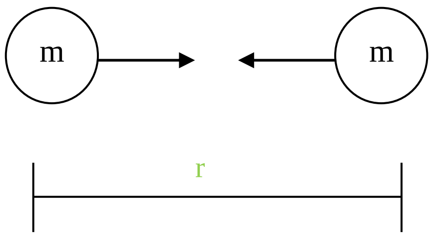
Gravitation law of attraction

States that any two bodies in the universe attract each other with a force that is directly forces proportional to the product of their masses and inversely proportional to the square of the distance between them.



$$F \,\alpha\, \frac{m_1 m_2}{r^2}$$

$$F = G\left(\frac{m_1 m_2}{r^2}\right)$$

G – Universal Gravitational constant

G =6.673 x 10^{-11} Nm^2/kg^2

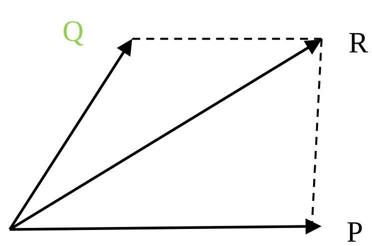
G value – Henry Cavendish – After Newton’s death

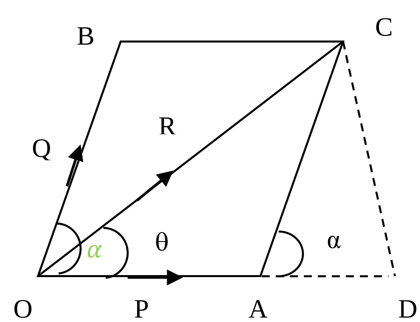
Earth’s standard acceleration due to gravity g = 9.80665 m/s^2 (32.1740 ft/s^2)

An object falling near the earth’s surface increases its velocity by 9.80655 m/s for each second of its descent.

Parallelogram law of forces

When two forces acting simultaneously at a point, can be expressed in both magnitude and direction by the two close sides of parallelogram drawn on a point, resultant is expressed completely, both in direction and magnitude by the diagonal of the parallelogram going through the point.

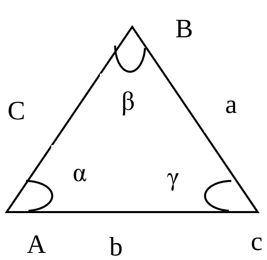




Sine law:

α - Angle b/w two forces

θ - Angle of resultant with x-axis



$$\frac{a}{Sin\,\alpha} = \frac{b}{Sin\,\beta} = \frac{c}{Sin\,\gamma^2}$$

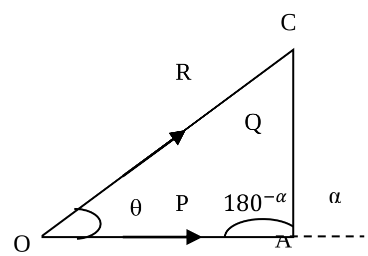
Corrine Law:

$a^2 = b^2 + c^2 - 2bc \cos\alpha$

$b^2 = a^2 + c^2 - 2ac \cos\beta$

$c^2 = a^2 + b^2 - 2ab \cos\gamma$

Direction of Resultant



$\lfloor \theta CD$ $\tan\theta = \frac{CD}{OD} = \frac{Q\,Sin\,\alpha}{P + Q\cos\alpha}$

$\theta = \tan^{-1}\left(\frac{Q\,Sin\,\alpha}{P + Q\cos\alpha}\right)$

Magnitude of Resultant (R)

$\lfloor DAC = \lfloor DOB = \alpha$, $AC = Q$

In triangle ACD

$AD = AC \cos\alpha = Q \cos\alpha$

$CD = AC\, Sin\,\alpha = Q \sin\alpha$

In triangle OCD

$OC^2 = OD^2 + OC^2$

$OC = R$, $OD = OA + AD$

$= P + Q\cos\alpha$

$$\therefore \quad OC^2 = OD^2 + OC^2$$

$$R^2 = (OA + AD)^2 + DC^2$$

$$= OA^2 + AD^2 + 2OA.AD + DC^2$$

$$R^2 = P^2 + Q^2\cos^2\alpha + 2\,PQ\cos\alpha + Q^2\sin^2\alpha$$

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha} \qquad (\therefore \sin^2\alpha + \cos^2\alpha = 1)$$

Case 1: If two forces P and Q acts at right angles, then

$\alpha = 90^\circ$

We know, magnitude of resultant.

$$R = \sqrt{P^2+Q^2+2PQ\cos\alpha} = \sqrt{P^2+Q^2+2PQ\cos 90°}$$

$$R = \sqrt{P^2+Q^2} \qquad [\therefore \cos 90 = 0]$$

We know, direction of resultant

$$\theta = \tan^{-1}\left(\frac{Q\ Sin\ \alpha}{P+Q\ \cos\ \alpha}\right) = \tan^{-1}\left(\frac{Q\ Sin\ 90^{\circ}\ 1}{P+Q\cos\ 90^{0}0}\right)$$

$$\theta = \tan^{-1}\left(\frac{Q}{P}\right)$$

Case 2: The two forces P & Q are equal and are acting at an angle α between them. (P = Q).

$$R = \sqrt{P^2+Q^2+2PQ\cos\ \alpha}$$

$$= \sqrt{P^2+P^2+2P^2\cos\ \alpha}$$

$$= \sqrt{2P^2(1+\cos\alpha)} \qquad \left(\therefore 1+\cos\alpha = 2\cos^2\frac{\alpha}{2}\right)$$

$$= \sqrt{2P^2\,2\cos^2\frac{\alpha}{2}} = \sqrt{4P^2\cos^2\alpha/2}$$

$$R = 2\ P\cos\frac{\alpha}{2}$$

$$Q = \frac{\alpha}{2}$$

Problem 1: The resultant of the two forces, when they act at an angle of 60° is 14N. If the same forces are acting at right angles, their resultant is $\sqrt{136}\,N$. Determine the magnitude of the two forces.

Soln.:

Case 1	**Case 2**
$R_1 = 14N$	$R_2 = \sqrt{136}\,N$

$\alpha_1 = 60^{\circ}$ $\alpha_2 = 90^{\circ}$

For case 1

R = $\sqrt{P^2 + Q^2 + 2PQ\ Cos\ \alpha}, 14 = \sqrt{P^2 + Q^2 + 2PQ\ Cos\ 60^{\circ}}$ 1/2

14 = $\sqrt{P^2 + Q^2 + PQ}$

196 = $P^2 + Q^2 + PQ$ → (1)

For case 2

R = $\sqrt{P^2 + Q^2}$ *or* $\sqrt{136} = \sqrt{P^2 + Q^2}$

136 = $P^2 + Q^2$ → (2)

(1) – (2) => 196-136 = $P^2 + Qx + PQ - P^2 - Q^2$

60 = PQ → (3)

(3) x 2 => 120 = 2 PQ → (4)

(4) + (2) => 136 + 120 = $P^2 + Q^2 + 2PQ$

256 = $P^2 + Q^2 + 2PQ$

$(16)^2$ = $(P\&Q)^2$

P+Q = 16

P = 16 – Q → (5)

Substitute (5) in (3)

60 = (16-Q) Q

60 = $16Q - Q^2$

$Q^2 - 16Q + 60 = 0$

This is a quadratic equation, so

$$Q = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

a = 1
b = -16
c = 60

$$= \frac{16 \pm \sqrt{256 - 240}}{2}$$

$$= \frac{16 \pm \sqrt{16}}{2} = \frac{16 \pm 4}{2} \therefore Q = 10\&6$$

$$= \frac{16 \pm \sqrt{16}}{2} = \frac{16 \pm 4}{2} \quad \therefore Q = 10 - 6$$

If Q = 10, P = 6

Q = 6, P = 10

Two forces are 10 N & 6 N