Engineering Mechanics

Mechanics:

Branch (or) study of bodies at rest or in motion when subjected to external mechanical disturbances.

Engineering Mechanics
- Mechanics of solids
  - Deformable bodies
  - Rigid bodies
    - Statics (Body at rest)
    - Dynamics (Body at motion)
      - kinematics
      - kinetics
- Mechanics of fluids
  - Statics
  - Dynamics
    - kinematics
    - kinetics

Statics:

↳ Deals with study of body at rest. Statics is the study of equilibrium of body under the action of force.

→ statics is mainly concerned with the conditions of equilibrium of stationery bodies.

Ex: support rnx of stationey body (beam) subjected to some external loads.

Dynamics:

↳ mainly concerned with the study of motion of body and the effect of force acting on them.

Ex: Force hit by a bat on ball.

Kinematics → branch of dynamics deals with the relationship b/w displacement, velocity, acceleration and time for given force. [without considering the ~~motion~~ force)

Kinetics:

↳ deals with relationship b/w the force acting on a body, the mass of body, ~~and~~ mass of body and motion of body. [with considering force).

Dimensions:

Primary Dimensions → Length → ~~mm~~ mm or cm or m, Mass → ~~kg~~ kg or ~~g~~, Time → sec

2° Dimensions →

Length L | Time → T

Area L^2

Volume L^3

Mass M

Velocity LT^{-1}

Acceleration → LT^{-2}

Angular velocity T^{-1}

Angular acceleration T^{-2}

Force → MLT^{-2}

Units:

(i) FPS system of units (Foot - Pound - Sec)

(ii) CGS system (cm, g, sec)

(iii) MKS system (m, kg, sec)

(iv) SI (International units).

1 kgf = 9.81 N

W = M × g (newton).

Mugll

Laws of Mechanics:

1) Newton First law
2) Newton second law
3) Newton third law
4) Gravitional Law of attraction
5) Parallelogram law of forces
6) Sine law
7) Cosine law
8) Lami's theorem
9) Principle of transmissibility of force.

Newton's First law:

A body remains in its state of rest or motion unless it is acted by external force to change its state.

Newton's second law:

Rate of change of momentum of a body is directly proportional to the force applied on the body and change of momentum takes place in the direction of force applied.

$$\boxed{F = ma}$$

Newton's 3rd law:-

For Every action there is equal and opposite reaction.

Gravitonal law of attraction:

This law states that two particles of mass m_1 and m_2 are attracted towards each other along the line connecting them with a force whose magnitude 'F' is proportional to the product of their masses and $\frac{1}{\alpha}$ to the sq of the distance b/w them.

$$F = G\left[\frac{m_1 m_2}{r^2}\right]$$

Parallelogram law of forces :

If two forces (F_1 and F_2) acting at a point to be represented in mag and direction by two adjacent sides of parallelogram, then their resultant is represented in magnitude and direction by the diagonal of the parallelogram at that point.

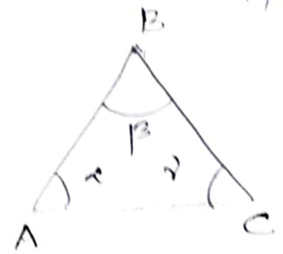
$$\vec{F} = \vec{F_1} + \vec{F_2} = \vec{F_2} + \vec{F_1}$$

If two forces acting simultaneously at a point be represented in magnitude and direction by two adjacent sides of a parallelogram then the resultant of these two forces is represent in magnitude and direction by the diagonal of the parallelogram originates from that point.

$$R = \sqrt{P^2 + Q^2 + (2PQ\cos\theta)}$$

Sine law :

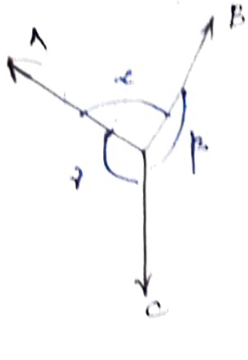
Triangle ABC with sides a, b, c and included angles α, β, γ as shown where $\alpha, \beta, \gamma \neq 90°$



$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

Lami's Theorem :

If 3 forces acting at a point are in equilibrium, each force will be proportional to the sine of the angle b/w the other two forces.

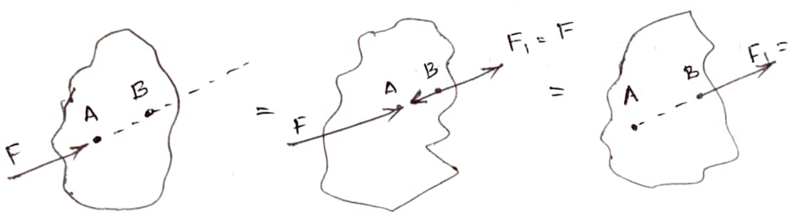


$$\frac{A}{\sin\beta} = \frac{B}{\sin\gamma} = \frac{C}{\sin\alpha}$$

$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}.$$

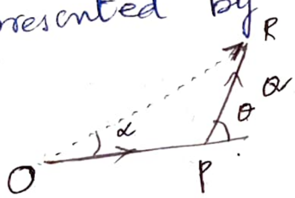
Principle of transmissibility of force

The state of rest or motion of a rigid body is unaltered if a force acting on the body is replaced by another force of the same magnitude and direction but acting anywhere on the body along the line of action of the replaced force.



Triangle law of forces:

If two forces acting at a point are represented by the two sides of triangle taken in order, then their resultant force is represented by third side taken in opposite order.



Scalar and Vector Quantities:

Scalar → Quantities which possess magnitude only

Ex: Length, area, Volume, mass etc.

Vector → Quantities which posses magnitude and direction

Ex: Force, Velocity, acceleration etc.

Vectorial Representation of Force and Moments

Vector → Mathematical expression possessing magnitude and direction.

$\vec{P}$

Vector Operations :

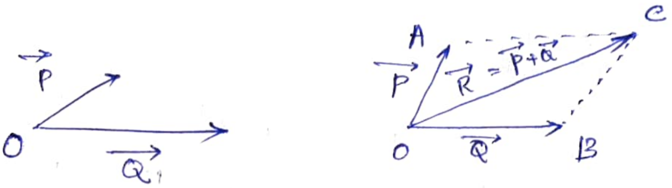
Addition of two Vectors :

→ Followed by two laws

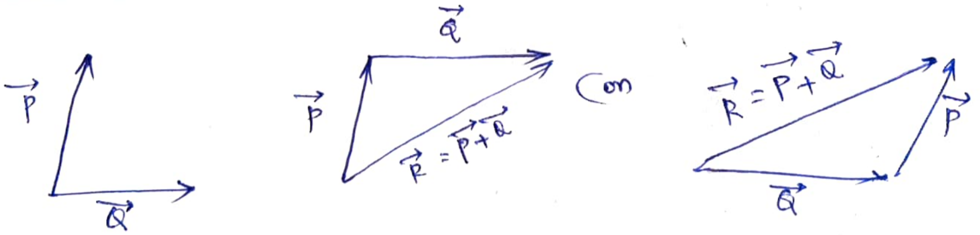
(a) Parallelogram law

(b) Triangle law.

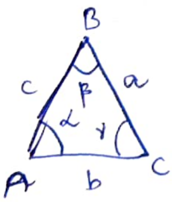
(a) Parallelogram law :-



(b) Triangle law :



Cosine law :



$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$

$$b^2 = c^2 + a^2 - 2ca\cos\beta$$

$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$

(P) A Force F has the components $F_x = 250\ N$ $F_y = -350$ $F_z = 500\ N$. Determine the magnitude of $\vec{F}$ and the angles θ_x, θ_y and θ_z that forms with the x, y and z axes.

$F_x = 250\,N \quad F_y = -350\,N \quad F_z = 500\,N$

$|\vec{F}| = F = \sqrt{F_x^2 + F_y^2 + F_z^2} = \sqrt{250^2 + (-350)^2 + 500^2}$

$= 659.54\,N.$

$\cos\theta_x = \frac{F_x}{F} = \frac{250}{659.54} = 67.73^\circ$

$\cos\theta_y = F_y / F = 122.05^\circ \quad \theta_z = \frac{F_z}{F} = 40.7^\circ$

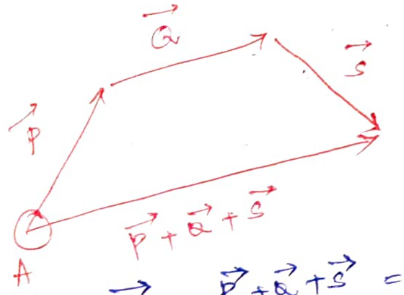
Zero Vector :- (Null Vector)

→ A zero Vector is obtained when a vector is subtracted from itself i.e. $\vec{A} - \vec{A} = \vec{0}$, where $\vec{A}$ is any vector

Composition of Vectors :

→ It is the process of determining the resultant of a system vector.

[tip-to-tail fashion and connecting the tail of the first vector with the tip of last one gives resultant].



$\vec{R} = \vec{P} + \vec{Q} + \vec{S} = (\vec{P} + \vec{Q}) + \vec{S} = \vec{P} + (\vec{Q} + \vec{S}) = (\vec{P} + \vec{S}) + \vec{Q}$

Resolution of Vector : → Opposite action of addition of vectors.

Unit Vector :

Vector $\vec{F}$ may be expressed by multiplying its magnitude F by a unit vector $\hat{n}$ whose magnitude is one and whose direction concides with that of $\vec{F}$.

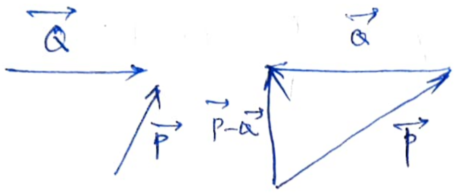
$$\vec{F} = F\hat{n}$$

$$\hat{n} = \frac{\vec{F}}{|\vec{F}|} = \frac{\vec{F}}{F}$$

Subtraction of Vector :

↳ addition of the corresponding negative vector

$$\vec{P} - \vec{Q} = \vec{P} + (-\vec{Q}).$$



Multiplication of Vector by scalars :

$$(m+n)\vec{P} = m\vec{P} + n\vec{P}$$

$$m(\vec{P}+\vec{Q}) = m\vec{P} + m\vec{Q}$$

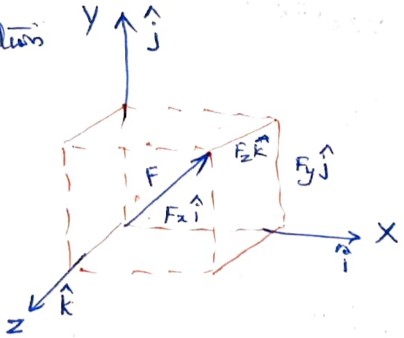
$$m(n\vec{P}) = n(m\vec{P}) + mn(\vec{P})$$

Representation of Vector in 3D

F_x ⇒ Force Components in x direction

F_y → " " " y dir

F_z → " " " z dir



$\hat{i}$ = unit vector in x dir

$\hat{j}$ = " " " y dir

$\hat{k}$ = " " " z dir.

Sum ⇒

$$\vec{F} = F_x\,i + F_y\,j + F_z\,k$$

magnitude $F = |\vec{F}| = \sqrt{F_x^2 + F_y^2 + F_z^2}$

Position Vector :

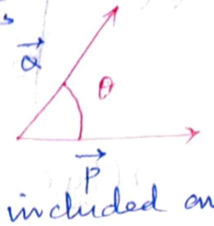
A position vector defines the position of a point in any coordinate system

$$\vec{r} = x\,i + y\,j + z\,k$$

$$r = \sqrt{x^2 + y^2 + z^2}$$

Dot (or) Scalar Product:

The dot (or) scalar product of two vectors $\vec{P}$ and $\vec{Q}$ is a scalar quantity and is defined as the product of the magnitude of two vectors and the cosine of their included angle θ



$$\vec{P}\cdot\vec{Q} = PQ\cos\theta$$

$0^\circ \le \theta \le 90^\circ \Rightarrow$ product is +ve

$90^\circ \le \theta \le 360^\circ \Rightarrow$ " " −ve

$$\vec{P}\cdot\vec{Q} = \vec{Q}\cdot\vec{P}$$

$$\vec{P}\cdot(\vec{Q}+\vec{R}) = (\vec{P}\cdot\vec{Q}) + (\vec{Q}\cdot\vec{R})$$

$$m(\vec{A}\cdot\vec{B}) = (m\vec{A})\cdot\vec{B} = \vec{A}\cdot m\vec{B} = \vec{A}\cdot\vec{B} = AB\cos\theta.$$

Since the unit vectors i, j, k are orthogonal

$$i\cdot j = j\cdot k = k\cdot i = (1)(1)\cos 90^\circ = 0$$

$$i\cdot i = j\cdot j = k\cdot k = (1)(1)\cos 0 = 1$$

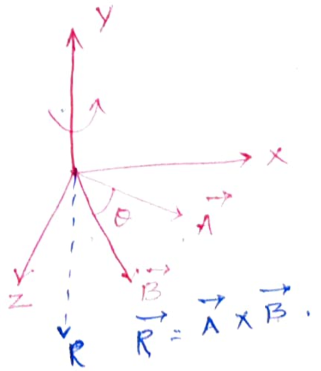
If $\vec{A} = A_x i + A_y j + A_z k$ and $\vec{B} = B_x i + B_y j + B_z k$ then

$$\vec{A}\cdot\vec{B} = A_x B_x + A_y B_y + A_z B_z$$

$$\vec{A}\cdot\vec{A} = A_x^2 + A_y^2 + A_z^2.$$

Cross (or) Vector Product:

The cross (or) vector product of two vectors $\vec{P}$ and $\vec{Q}$ is defined as the product of magnitude of the two vector and the sine of their included angle. The resultant vector is respresented by $\vec{R}$



$$\vec{R} = \vec{A}\times\vec{B} = (AB\sin\theta)\hat{n}$$

$\vec{P}\times\vec{Q} \neq \vec{Q}\times\vec{P}$ (not commutative)

$$\vec{P}\times\vec{Q} = -\vec{Q}\times\vec{P}$$

If $\vec{P} = P_x i + P_y j + P_z k$

$\vec{Q} = Q_x i + Q_y j + Q_z k$

$$\vec{P}\times\vec{Q} = \begin{vmatrix} i & j & k \\ P_x & P_y & P_z \\ Q_x & Q_y & Q_z \end{vmatrix}$$

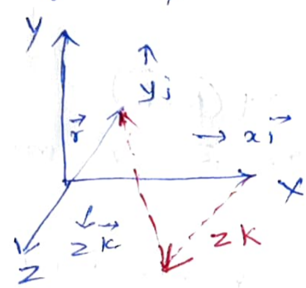
$$\vec{P}\times\vec{Q} = i(P_yQ_z - Q_yP_z) - j(P_xQ_z - Q_xP_z) + k(P_xQ_y - Q_xP_y)$$

Position Vector.

↳ defines the position of a point in any coordinate system.

$$\vec{r} = xi + yj + zk$$

$$r = \sqrt{x^2+y^2+z^2}$$



Problem:

① Find the vector $(-3\vec{A} - 2\vec{B} - \vec{C})$ in terms of i, j, k and its magnitude $\vec{A} = -i + 2j - k$; $\vec{B} = -3i+4j-3k$

$$\vec{C} = 5i - 2j + 4k$$

$$(-3\vec{A} - 2\vec{B} - \vec{C}) = -3(-i+2j-k) - 2(-3i+4j-3k)$$
$$\cancel{(5)} - (5i-2j+4k)$$

$$= i(3+6-5) + j(-6-8+2) + k(3+6-4)$$

$$= 4i - 12j + 5k$$

$$\text{magnitude} = \sqrt{4^2+(-12)^2+5^2} = 13.6.$$

② Determine the unit vector along the line which orginates at point (4, 1, −2) and passes through the point (2, 2, 6)

$\vec{OA} = (x_1, y_1, z_1) = (4, 1, -2)$ $\vec{OB} = (x_2, y_2, z_2) = (2, 2, 6)$

Soln:

Let the starting point of line be A and the other point through which the line passes be B.

The unit vector, $\hat{n} = \dfrac{\vec{AB}}{|\vec{AB}|} = \dfrac{\vec{AB}}{AB}$

$\vec{AB} = \vec{OB} - \vec{OA} = (x_2 - x_1)i + (y_2 - y_1)j + (z_2 - z_1)k$

$= (2-4)i + (2-1)j + [6-(-2)]k =$

$-2i + j + 8k$

$AB = \sqrt{(-2)^2 + 1^2 + 8^2} = 8.3$ $\hat{n} = \dfrac{-2i + j + 8k}{8.3}$

Find the following.

If $\vec{A} = 4i + 8j - 14k$ $\vec{B} = 6i - 3j - 2k$

(i) $2\vec{A} + 5\vec{B}$

$2\vec{A} = 2(4i + 8j - 14k) = 8i + 16j - 28k$

$5\vec{B} = 5(6i - 3j - 2k) = 30i - 15j - 10k$

$2\vec{A} + 5\vec{B} = 38i + j - 38k$

(ii) $2\vec{A} \cdot 3\vec{B}$

$2\vec{A} \cdot 3\vec{B} = 2(4i + 8j - 14k) \cdot 3(6i - 3j - 2k)$

$= (8i + 16j - 28k) \cdot (18i - 9j - 6k)$

$= 144 - 144 + 12 = 12$ $\rightarrow 168$ $\rightarrow 168$

(iii) $3\vec{A} \times 4\vec{B}$

$3\vec{A} = 3(4i + 8j - 14k)$ $4\vec{B} = 4(6i - 3j - 2k)$

$$3\vec{A} \times 4\vec{B} = \begin{vmatrix} i & j & k \\ 12 & 24 & -42 \\ 24 & -12 & -8 \end{vmatrix}$$

$= -696i - 912j - 720k.$

(iv) $(2\vec{A} \times 3\vec{B}) \cdot (\vec{A} \times 2\vec{B})$

$2\vec{A} = 8i + 16j - 28k$ $\quad 3\vec{B} = 18i - 9j - 6k$

$$2\vec{A} \times 3\vec{B} = \begin{vmatrix} i & j & k \\ 8 & 16 & -28 \\ 18 & -9 & -6 \end{vmatrix} = -348i - 456j - 360k$$

$$\vec{A} \times 2\vec{B} = \begin{vmatrix} i & j & k \\ 4 & 8 & -14 \\ 12 & -6 & -4 \end{vmatrix} = -116i \; \overset{(-)}{\oplus} 152j - 120k$$

$$(2\vec{A} \times 3\vec{B}) \cdot (\vec{A} \times 2\vec{B}) = (-348i - 456j - 360k)$$
$$(-116i \overset{(-)}{+} 152j - 120k)$$

$$= 40368 + 693.12 + 43200 = 152880.$$

(P) Force $\vec{F} = 6i - 3j - 2k$ act at a point $P(1, 3, 4)$. Determine the moment of this force about the point of origin.

348
116

Given:

$P(1, 3, 4)$

→ Position vector $\vec{r} = i + 3j + 4k$

Moment of force, $\vec{M} = \vec{r} \times \vec{F}$

$$= \begin{vmatrix} i & j & k \\ 1 & 3 & 4 \\ 6 & -3 & -2 \end{vmatrix} = 6i + 26j - \cancel{24} \, 21k$$

* When two vectors are at right angles to each other the dot product of vector should be zero.

* For Parallel vectors, their cross product should be "zero"

System of Force :

Force :

Force is an action exerted on a body which changes or tends to change the state of rest or of uniform motion of the body. Force is vector quantity.

Characteristics of Force

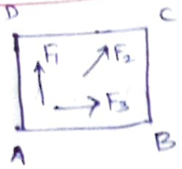
(i) Magnitude → value
(ii) Line of action
(iii) Direction.

System of force → A body with two (or) more faces acting simultaneously on its constitute a system of forces.

System of force

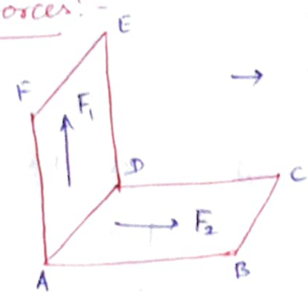
- Coplanar
  - Collinear
    - like
    - unlike
  - Concurrent
  - Non-concurrent
    - Parallel
      - like
      - unlike
    - Non-parallel
- Non-Coplanar
  - Concurrent
  - Non-concurrent
    - Parallel
      - like
      - Unlike
    - Non parallel

Coplanar Force



→ All forces acting in one plane.

Non-Coplanar Forces:-



→ the forces do not act in one plane. Line of action of Force F_2 lies in ABCD plane, but line of action of force F_1 lies in ADEF plane.

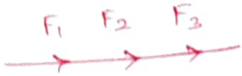
The system is also called as "Forces in space"

Collinear Force:

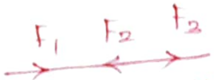
↳ Forces which acts on a common line of action are

If they act in same direction → like Collinear

If they act in opposite " → Unlike Collinear.



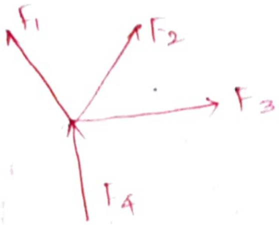
Like Collinear



Unlike Collinear.

Concurrent Force:

Force intersects at a common point.



Parallel Force:

In parallel force system, line of action of forces are parallel to each other. Parallel forces acting in same direction are called like parallel forces and the parallel forces acting in opposite direction are called unlike parallel forces.

like Parallel

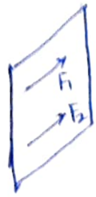
Unlike Parallel.

Like Collinear Coplanar forces:

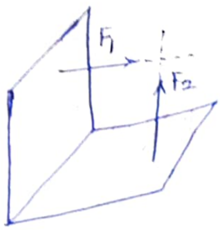
Unlike Collinear coplanar forces:

Coplanar concurrent forces: ⟶

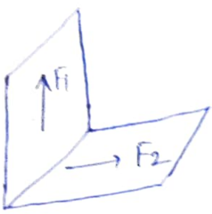
Coplanar non-concurrent forces ⟶



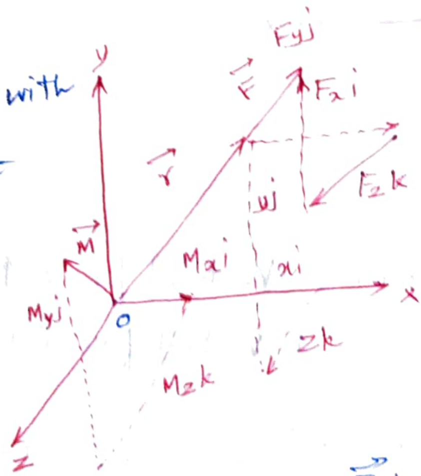
Non-coplanar concurrent forces:



Non-coplanar and Non-concurrent forces:



Moment of a Force:



The moment $\vec{M}$ of a force $\vec{F}$ with respect to point 'o' is the cross product

$\vec{M} = \vec{r} \times \vec{F}$

$\vec{r}$ is the position vector relative to point 'o' of any point P on the line of action of force $\vec{F}$.

→ The moment $\vec{M}$ represents the tendency of force $\vec{F}$ to rotate the body on which it acts about any axis, which passes through 'o' and ⊥r to the plane containing the $\vec{F}$ and $\vec{r}$

$$= \vec{r} \times \vec{F}$$

$$\vec{r} = xi + yj + zk$$

$$\vec{F} = F_x i + F_y j + F_z k$$

$$\vec{M} = M_x i + M_y j + M_z k$$

$$\vec{M} = \vec{r} \times \vec{F} = \begin{vmatrix} i & j & k \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix}$$

$\vec{M}$ along x axis

$$= \vec{M} \cdot i$$

$$= (M_x i + M_y j + M_z k) \cdot (i)$$

$$= M_x(1) + M_y(0) + M_z(0)$$

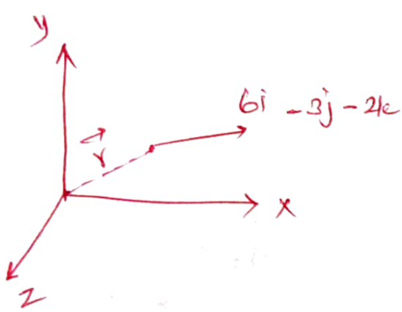
$$= M_x .$$

(P) A Force $\vec{F} = 6i - 3j - 2k$ acts at a point $P(1,3,4)$ Determine the moment of this force about the pt of origin.

$P(1,3,4)$

$$\vec{r} = i + 3j + 4k$$

$$\vec{M_o} = \vec{r} \times \vec{F} = \begin{vmatrix} i & j & k \\ 1 & 3 & 4 \\ 6 & -3 & -2 \end{vmatrix} = 6i + 26j - 21k .$$



Force :

- Upward Vertical force (↑ +)
- Downward Vertical force (↓ −)
- Horizontal force (→ +) (L to R)
- Horizontal force (← −) (R to L).

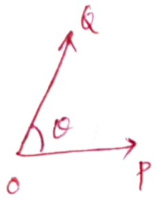
Resultant force of two Concurrent forces : (Analytical)

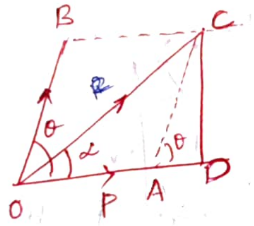
Parallelogram law of forces :

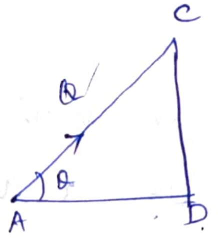
If two forces acting simultaneously at a point be represented in magnitude and direction by the two adjacent sides of a parallelogram, then the resultant of these two forces is represented in magnitude and direction by the diagonal of that parallelogram originating from that point.

Proof :

P and Q are two concurrent forces acting on point 'O' at an angle of θ as shown. The forces P and Q are graphically represented by the lines OA and OB respectively. The parallelogram OA$\hat{C}$B is completed by drawing the lines BC and AC parallel to OA and OB resp. In OACB, the diagonal OC represents the resultant forces of P and Q.







In Δ ACD

$$\cos\theta = \frac{AD}{Q} \qquad \sin\theta = \frac{CD}{Q}$$

Ac

$$AD = Q\cos\theta \rightarrow (1) \qquad CD = Q\sin\theta \rightarrow (2)$$

$$AD^2 + CD^2 = AC^2 = Q^2 \Rightarrow (3)$$

Δ OCD

$$OC^2 = OD^2 + CD^2$$

$$= (OA + AD)^2 + CD^2$$

$$= OA^2 + AD^2 + 2 \cdot OA \cdot AD + CD^2$$

$$OC^2 = OA^2 + (AD^2 + CD^2) + 2 \cdot OA \cdot AD$$

$$OC^2 = OA^2 + AC^2 + 2 \cdot OA \cdot AD \text{ Con}$$

$$R^2 = P^2 + Q^2 + 2PQ\cos\theta$$

$$\therefore R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$

Inclination of Resultant force R.

In Δ OCD

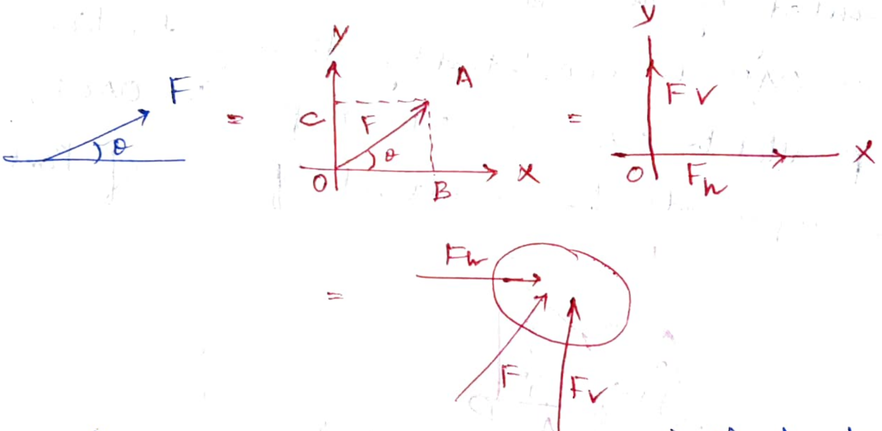
$$\tan\alpha = \frac{CD}{OD} = \frac{Q\sin\theta}{P+Q\cos\theta} =$$

Resolution of a Force :

→ Force is either horizontal (or) vertical.

→ Horizontal force may be either LHS (or) RHS. direct

Splitting up a force into components along the fixed reference axes is called Resolution of a force.



Consider an inclined force F, inclined at angle θ with the horizontal as shown above. Let us resolve the force (or split) into 2 components along two fixed axes. (i.e) OX and OY.

Magnitude of components:-

Let F_h = Horizontal component of force F

= OB (or) CA

F_v = Vertical Component of force F

= OC (or) BA

Consider the right angled Δ^{le} OAB

$$\cos\theta = \frac{OB}{OA} = \frac{F_h}{F} \Rightarrow F_h = F\cos\theta$$

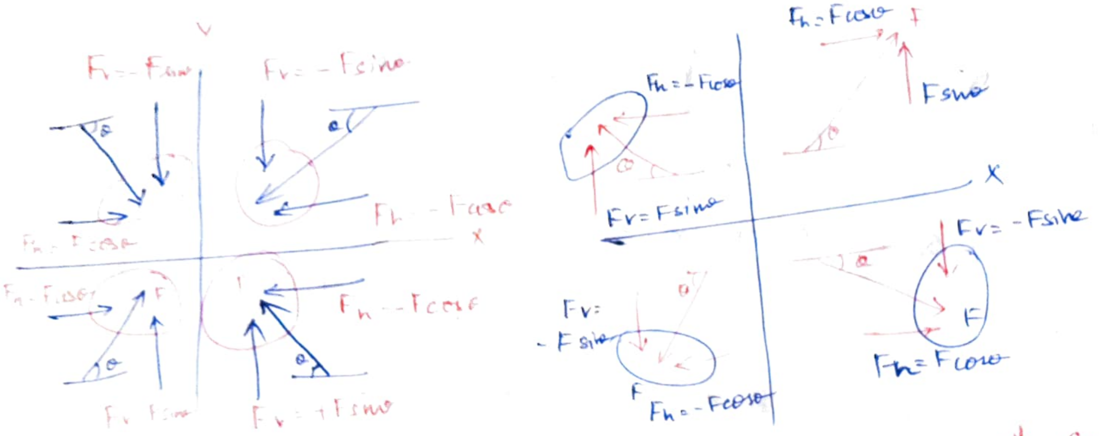
$$\sin\theta = \frac{AB}{OA} = \frac{F_v}{F} \Rightarrow F_v = F\sin\theta$$

$\Rightarrow$ θ is the inclination of a force F ~~with~~ w.r.to x axis, then magnitude of its hor ~~to~~ or vertical components are $F\cos\theta$ and $F\sin\theta$.

Direction of the component

F_h - Horizontal components $= +F\cos\theta$

F_v - Vertical Components $= +F\sin\theta$.



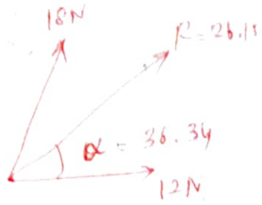
(P) Two concurrent forces of 12N and 18N are acting at an angle 60°. Find Resultant

$P = 12N \quad Q = 18N \quad \theta = 60°$

$$R = \sqrt{P^2+Q^2+2PQ\cos\theta} = \sqrt{12^2+18^2+2(12)(18)\cos 60°}$$

$$= 26.15N$$

Inclination of Resultant force with force P



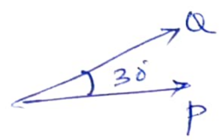
$$\tan\alpha = \frac{Q\sin\theta}{P+Q\cos\theta}.$$

$$\tan\alpha = \frac{18\sin 60^\circ}{12 + (18\cos 60^\circ)} = 0.742$$

$$\alpha = \tan^{-1}(0.742) = 36.57$$

(P) Two concurrent forces act an angle of 30°. The resultant force is 15N and one of the force is 10N. Find the other force.

$R = 15N \quad P = 10N \quad \theta = 30^\circ \quad Q = ?$



$$R^2 = P^2 + Q^2 + 2PQ\cos\theta$$

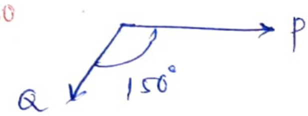
$$15^2 = 10^2 + Q^2 + 2(15)(Q)\cos 30^\circ$$

$$Q^2 + 17.32\,Q - 125 = 0$$

$$Q = \frac{-17.32 \pm \sqrt{(17.32)^2 - (4 \times 1 \times -125)}}{2 \times 1} = \frac{-17.32 \pm 28.28}{2}$$

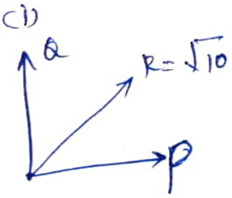
$$Q = -22.8N \text{ or } 5.48N$$

Rejects $-22.8N$ force because if the force is reverse the include angle will be 150° not 30°



(P) Find the magnitude of two forces, such that If they act at right angles, their resultant is $\sqrt{10}N$, But if acts at 60°, their resultant is $\sqrt{13}N$.

case (i)

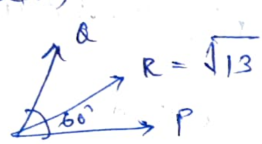


$$R^2 = P^2 + Q^2 + 2PQ\cos 90^\circ$$

$$R^2 = P^2 + Q^2$$

$$10 = P^2 + Q^2 \rightarrow (1)$$

case (ii)



$$R^2 = P^2 + Q^2 + 2PQ\cos 60^\circ$$

$$R^2 = P^2 + Q^2 + PQ$$

$$13 = P^2 + Q^2 + PQ$$

(2) in (1)

~~using $(P+Q)^2 = P^2 + Q^2 + 2PQ = 10^2 + (2\times 3) = 16$~~

$13 = 10 + PQ$

$PQ = 3$

Using $(P+Q)^2 = P^2 + Q^2 + 2PQ$

$= 10^2 + (2\times3)$

$= 16$

$P+Q = \sqrt{16}$ $\quad$ $P+Q = 4$

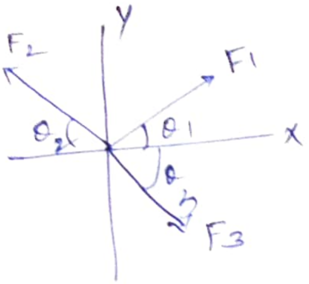
$(P-Q)^2 = P^2 + Q^2 - 2PQ$

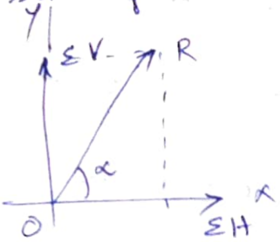
$= 10 - (2\times3) = 4$ $\quad$ $(P-Q) = 2 \rightarrow (4)$

~~$P+Q=4$~~ $\rightarrow (3)$ $\quad$ $P = 3N$ $\quad$ $Q = 1N$

Finding the resultant force of more than 2 concurrent force.

(1) Find the algebric sum of the horizontal components





Resolving the forces horizontally (i.e along x axis)

$\Sigma H = F_1 \cos\theta_1 - F_2 \cos\theta_2 + F_3 \cos\theta_3$

(2) The algebric sum of vertical components

$\Sigma V = F_1 \sin\theta_1 + F_2 \sin\theta_2 - F_3 \sin\theta_3$

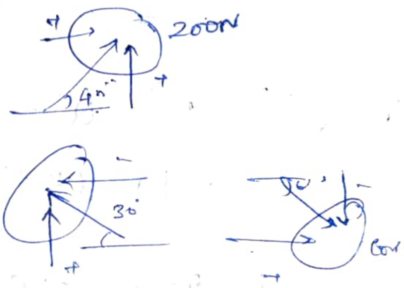
③ Find the magnitude of resultant force.

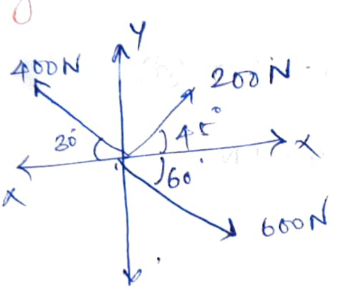
$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}.$$

④ Direction of Resultant force

$$\tan\alpha = \frac{\Sigma V}{\Sigma H} \quad \text{(or)} \quad \alpha = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right).$$

Ⓟ 3 Coplanar con current forces are acting at a point as shown. Determine the resultant in magnitude and direction.





Resolving forces

$\Sigma H = 200\cos 45^\circ - 400\cos 30^\circ + 600\cos 60^\circ$

$= 141.42 - 346.41 + 300 = 95.01N$

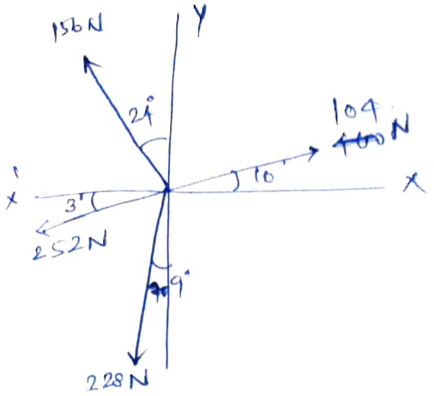
$\Sigma V = 200\sin 45^\circ + 400\sin 30^\circ - 600\sin 60^\circ$

$= 141.42 + 200 - 519.62 = -178.2N$

$R = \sqrt{(95.01)^2 + (-178.2)^2} = 201.95N.$

$\alpha = \tan^{-1}\left(\frac{178.2}{95.01}\right) = 61.93^\circ$

Ⓟ The 4 coplanar forces are acting at a point as shown. Determine the Resultant in magnitude & direction.



Soln:

$F_1 = 104\,N$

$F_2 = 156\,N$

$F_3 = 252\,N$

$F_4 = 228\,N$

Angle of inclination of forces 156 N and 228 N are given with reference to y axis, Hence θ to be found with x axis.

$\therefore\ \theta_1 = 10^\circ \quad \theta_2 = 66^\circ \quad \theta_3 = 3^\circ \quad \theta_4 = 81^\circ$

$\Sigma H = 104\cos 10^\circ - 156\cos 66^\circ - 252\cos 3^\circ - 228\cos 81^\circ$

$= -248.32\,N$

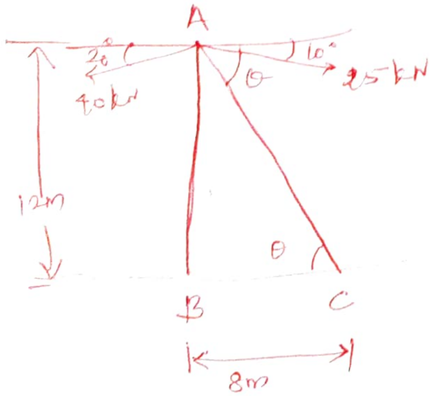
$\Sigma V = 104\sin 10^\circ + 156\sin 66^\circ - 252\sin 3^\circ - 228\sin 81^\circ$

$= -77.82$

$R = \sqrt{(-248.32)^2 + (-77.82)^2} = 260.2\,N$

$\alpha = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right) = \tan^{-1}\left(\frac{77.82}{248.32}\right) = 17.4^\circ$

(P) Two cables which have known tensions are attached to top of a tower AB. A third cable AC is used as a Guy wire as shown. Determine the tension AC. If the resultant of the forces exerted at A by the 3 cables acts vertically downward.



Soln:

It is given that the resultant force is acting vertically downwards. Hence the net horizontal force is zero. and the resultant force is equal to the net of vertical force.

$\triangle ABC$ $\tan\theta = \frac{12}{8}$

$\theta = 56.30^\circ$

$\Sigma H = 25\cos 10^\circ - 40\cos 20^\circ + T\cos\theta = 0$

$0 = 25\cos 10^\circ - 40\cos 20^\circ + T\cos 56.3^\circ$

$T = 23.35N$,

$\Sigma H = 0$ $\boxed{\Sigma V = R}$

Resultant force of two Concurrent force.

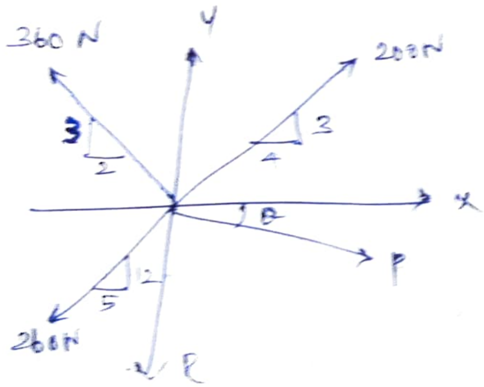
① Parallelogram law

② Δ^{le} law of forces.

Resultant force of more than two concurrent forces

① Polygon law.

Q. R of forces shown in fig. is 520N along the -ve direction of y axis. Find P and θ.



Given

$R = 520N$ acting along -ve direction of Y axis

$\Sigma H = 0$ $\Sigma V = R = -520N$.

$F_1 = 200N \quad \theta_1 = \tan^{-1}\left[\frac{3}{4}\right] = 36.87$

$F_2 = P$ angle is θ

$F_3 = 260N \quad \theta_3 = \tan^{-1}\left[\frac{12}{5}\right] = 67.38°$

$F_4 = 360N \Rightarrow \theta_4 = \tan^{-1}\left[3/2\right] = 56.31°$

$\Sigma H = 200 \cos 36.87 + P\cos\theta - 260\cos 67.38 - 360\cos 56.31$

$P\cos\theta = 139.69N \rightarrow (1)$

$\Sigma V = 200 \sin 36.87 - P\sin\theta - 260 \sin 67.38 + 360 \sin 56.31$

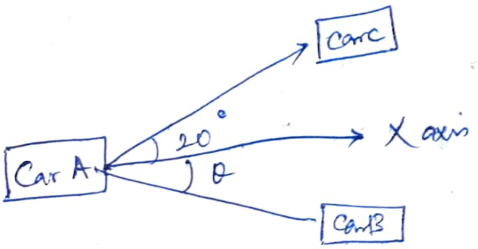
$P\sin\theta = 699.53 \rightarrow (2)$

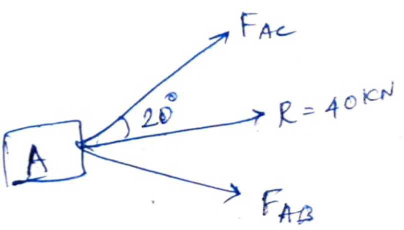
$\frac{P\sin\theta}{P\cos\theta} = \frac{699.53}{139.69} \Rightarrow \tan\theta = 5.007 \quad \boxed{\theta = 78.°7}$

$P\cos 78.7 = 139.69$

$P = 712.9N \quad \theta = 78.7$

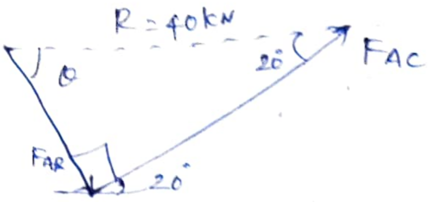
P) A car is pulled by means of 2 cars as shown. If the resultant of 2 forces acting on the car A is 40kN being directed along the +ve direction of X axis, det the θ of the cable attached to the car B, such that the force in cable AB is minimum. What is the magnitude of force in each cable when it occurs?

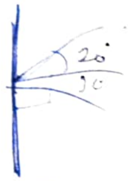




By Δ^{le} law of forces:

180
20





$\theta = (180 - 90 - 20) = 70°$

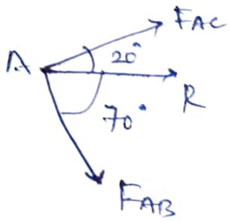
180 - 20 - 90
= 70

Force on cable

$$\frac{40}{\sin 90°} = \frac{F_{AB}}{\sin 90°} = \frac{F_{AC}}{\sin 70°}$$

$F_{AB} = 13.68\,kN$

$F_{AC} = 37.587\,kN$

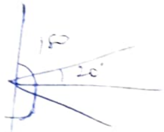


$\Sigma H = F_{AC} \cos 20° + F_{AB} \cos 70°$

$\Sigma H = 0.939\,F_{AC} + 0.342\,F_{AB}$ → (1)

$\Sigma V = F_{AC} \sin 20° - F_{AB} \sin 70°$

$= 0.342\,F_{AC} - 0.939\,F_{AB}$ → (2)



$\Sigma H = R = 40\,kN \qquad \Sigma V = 0$

$40 = 0.939\,F_{AC} + 0.342\,F_{AB}$

$0 = 0.342\,F_{AC} - 0.939\,F_{AB}$

$F_{AB} = 13.68\,kN \qquad F_{AC} = 37.87\,kN$

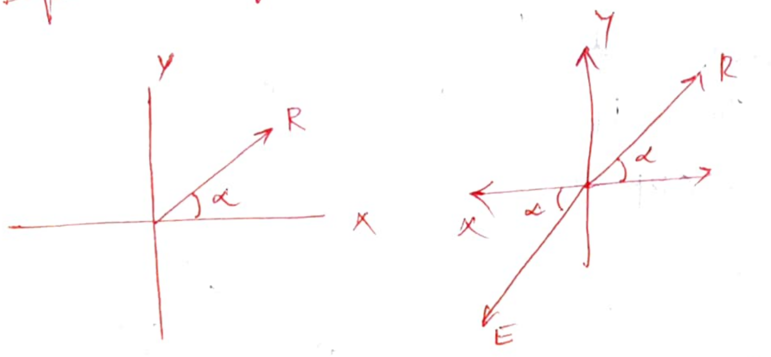
~~180 - 90~~ + 90 - θ

Equilibrium

The effect of resultant force on the particle is that, the particle starts moving in direction of resultant force. But, with the action of force, if the particle does not start moving or particles moves with uniform motion then the particles is in equilibrium.

* If the resultant of a number of forces acting on a particle is zero, the particle is in equilibrium,

* The set of forces, where resultant is zero is called Equilibrium forces.



R → resultant force, the particles starts to moves in the direction of R.

But if we apply an additional force of same magnitude and direction as that of resultant force, on the same line of action but in opposite direction, then the movement will be arrested (or) the particle is said to be in equilibrium.

The force E, which brings the particles. (or set of forces) to equilibrium is called as Equilibriant (E)

$\boxed{E = R}$ ⇒ Collinear but opp in nature.

Conditions of Equilibrium

$R = 0$ $\quad R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$

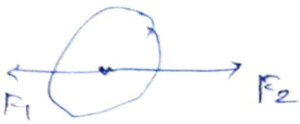
Both ΣH and ΣV are to be zero for equilibrium condition.

If $\Sigma V = 0$ $\Sigma H = 0$ ⇒ Then it is equilibrium.

Principle of Equilibrium

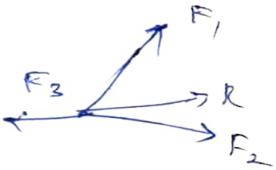
force

① 2 Principle .



If a body is subjected to two forces, then the body will be in equilibrium if the two forces are Collinear, equal and opposite.

② 3 Force equilibrium



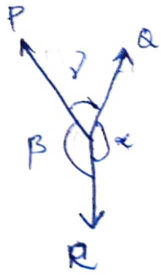
$R = F_3$ R is resultant of F_1 & F_2

Solution of equilibrium of coplanar forces:

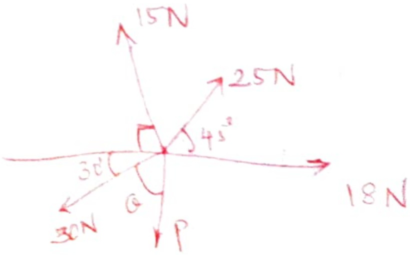
① Lami's theorem

If 3 coplanar forces acting at a point ~~is~~ be in equilibrium then each force is proportional to the sine of the angle between the other two.

$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$



P) The forces shown in equilibrium. Find the magnitude and direction of the unknown force P.



The inclined forces are resolved into 2 components in horizontal and vertical direction and the algebraic sum of horizontal components is equated to zero.

$\Sigma_H = 0 \; (\rightarrow +)$

$25 \cos 45^\circ + 18 - 30 \cos 30 - P \cos\theta = 0.$

$P\cos\theta = 9.7 \rightarrow (1)$

$\Sigma_V = 0 \; (\uparrow +)$

$15 + 25 \sin 45^\circ - 30 \sin 30^\circ - P \sin\theta = 0$

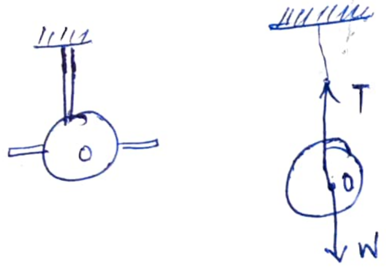
$P\sin\theta = 17.68.$

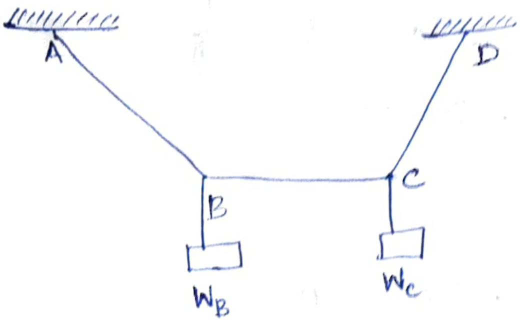
$\frac{P\sin\theta}{P\cos\theta} = \frac{17.68}{9.7}$ $\quad \theta = 61.24.$ $\quad P = 20.16N.$

Free body diagram.

→ In equilibrium analysis of structures/machines, it is necessary to consider all the forces acting on body and exclude all forces which are not directly applied on it. The problem becomes much simple if each body is considered in isolation (i.e) separate from the surrounding body (or) bodies. Such a body which has been so separated (or) isolated from the surrounding bodies is called Free body.

→ The sketch showing all the force (both external forces and reactions) and moments acting on a body is called Force body diagram.





Free body diagram at B.

Force acting at B are.

(i) weight of the body attached at B, acting downwards let it be W_B.

(ii) Tension on string AB, acting at B and towards A, let it be T_{BA}

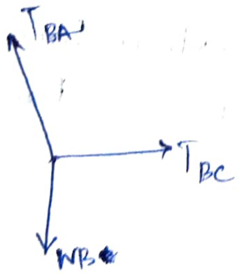
(iii) Tension on string BC, acting at B and towards C, let it be T_{BC}

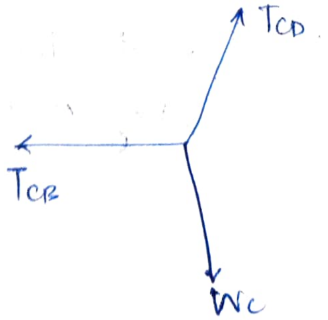
Free body diagram at C

(i) Weight of the body, attached at C, acting downwards let it be W_C

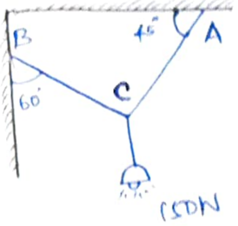
(ii) Tension on string CB, acting at C and towards B let it be T_{CB}.

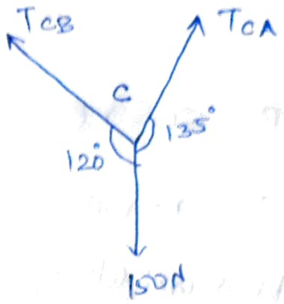
(iii) Tension on string CD, acting at C and towards D, let it be T_{CD}.





(P) An electric light fixture weighing 150N hangs from a point C, by two strings AC and BC as shown. Det the forces in the strings AC and BC.





Solve by Lami theorem at C

$$\frac{T_{CB}}{\sin 135} = \frac{T_{CA}}{\sin 120} = \frac{150}{\sin 105}$$

$T_{CB} = 109.81$ N $\quad T_{CA} = 134.49$ N.

By using polygon law

$\Sigma H = 0\ (\rightarrow) \rightarrow \quad T_{CA}\cos 45^\circ - T_{CB}\cos 30 = 0$

$T_{CA}\cos 45^\circ = T_{CB}\cos 30^\circ$

$T_{CA} = 1.244\, T_{CB} \rightarrow (1)$

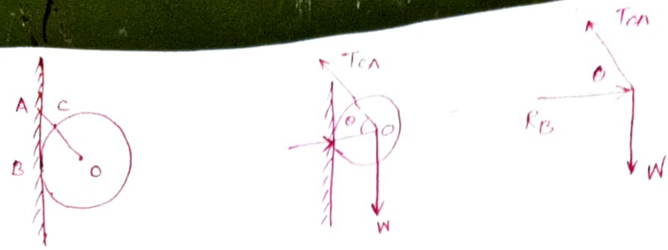
$\Sigma V = 0 \rightarrow \quad T_{CA}\sin 45 + T_{CB}\sin 30 - 150 = 0 \rightarrow (2)$

sub (1) in (2)

$0.866\, T_{CB} + 0.5\, T_{CB} = 150$

$T_{CB} = 109.81$ N $\quad T_{CA} = 134.49$ N

(P) A smooth sphere of weight W is supported by a string fastened to a point A on the smooth vertical wall, the other end is in contact with point B on the wall as shown. If the length of the string AC is equal to radius of the sphere, find the tension in the string and reaction of the wall.



<u>Given</u>

radius of sphere, $OB = OC = r$ [radius].

$AC = r = $ radius.

$W \Rightarrow$ weight of sphere.

In Triangle AOB

$OA = OC + CA = r + r = 2r$ $\quad \underline{OB = r}$

$$\cos\theta = \frac{OB}{OA} = \frac{r}{2r} = \frac{1}{2}$$

$$\underline{\theta = 60^\circ}$$

Apply $\Sigma H = 0$ at O.

$R_B - T_{CA}\cos\theta = 0 \rightarrow (1)$

$\Sigma V = 0$

$T_{CA}\sin\theta - W = 0$

$$T_{CA} = \frac{W}{\sin\theta} = \frac{W}{\sin 60^\circ} = 1.155W \rightarrow (2)$$

(2) in (1) $\quad R_B - 1.155W\cos 60^\circ = 0$

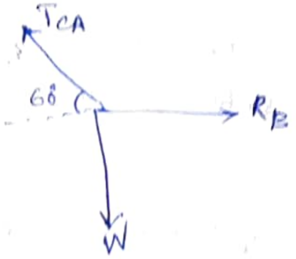
$R_B = 0.577W$

Tension of string $AC = 1.155W$

Reaction of wall $= 0.577W$.

By lami theorem

Angle b/w T_{CA} and $R_B = 120^\circ$

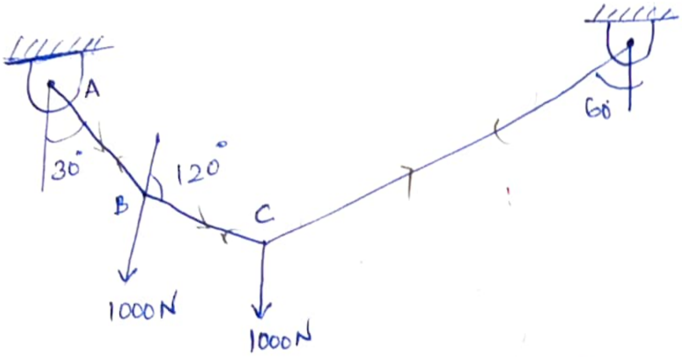


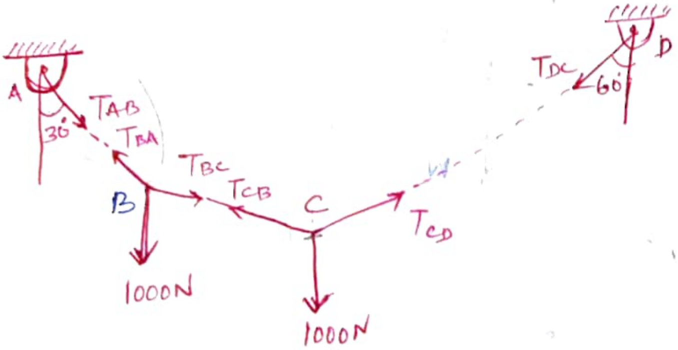
$$\frac{T_{CA}}{\sin 90^\circ} = \frac{R_B}{\sin 150} = \frac{W}{\sin 120}$$

$T_{CA} = 1.155W$; $R_B = 0.577W$.

(P) A string ABCD attached to two fixed points A and D has two equal weights of 1000N attached to it at B and C. The weighs rest with the portions AB and CD inclined at angles of 30° and 60° resp, to the vertical as shown. Find the tensions in the portions AB, BC and CD of the string, if the inclination of the portion BC with the vertical is 120°.

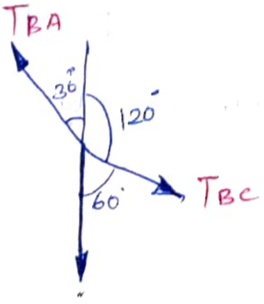
(S)





From the above diagram $T_{BA} = T_{AB}$; $T_{BC} = T_{CB}$; $T_{CD} = T_{DC}$.

Apply Lami Theorem at Joint B



Angle between T_{BA} & $T_{BC} = 30 + 120 = 150^\circ$

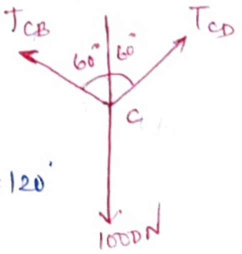
T_{BC} & 1000N is $180 - 120 = 60^\circ$

$$\frac{T_{BA}}{\sin 60^\circ} = \frac{T_{BC}}{\sin 150} = \frac{1000}{\sin 150}$$

$T_{BA} = 1732\ N$

$T_{BC} = 1000\ N$.

Lami Theorem at C



Angle b/w T_{CD} and $T_{CB} = 120^\circ$

" " T_{CD} and $1000\ N = 180 - 60 = 120^\circ$

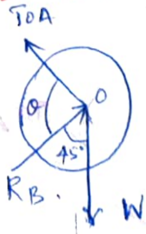
$$\therefore \frac{T_{CB}}{\sin 120^\circ} = \frac{T_{CD}}{\sin 120} = \frac{1000}{\sin 120}$$

$\boxed{T_{CD} = 1000\ N}$

$\boxed{T_{CB} = T_{BC} = 1000\ N}$

Tension AB = 1732 N
BC = 1000 N
CD = 1000 N.

(P) String AO holds a smooth sphere on an inclined plane ABC as shown. The weight of sphere is 1000 N and plane is smooth. Calculate the tension in the sling and the reaction at point of contact B.

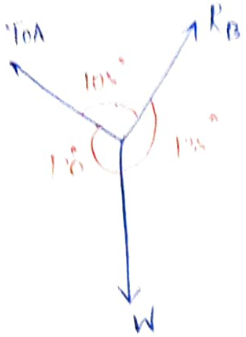


In right angle Δ^{le} OAB

OAB = 15°   ABO = 90°

$\theta = 75^\circ \Rightarrow \angle BOA = 180 - 90 - 15^\circ$

Angle b/w R_B & $W = 45^\circ$



T_{OA} & W $= 45° + 75° = 120°$

R_B & T_{OA} $= 180° - 75° = 105°$

R_B & W $= 360° - 120 - 105 = 135°$

Apply Lami's theorem at O.

$$\frac{T_{OA}}{\sin 135} = \frac{R_B}{\sin 120} = \frac{W}{\sin 105}$$

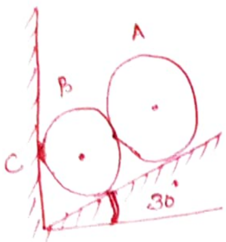
$$T_{OA} = \frac{W \sin 135°}{\sin 105} = \frac{1000 \sin 135°}{\sin 105} = 732\ N.$$

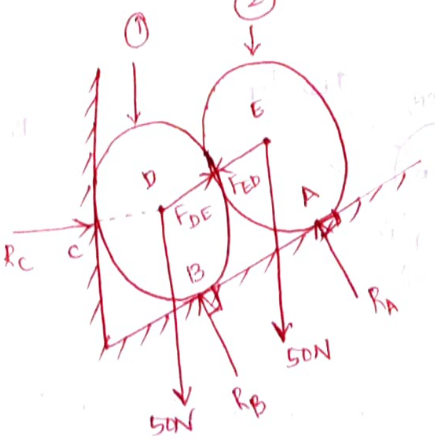
$$R_B = \frac{1000 \sin 120°}{\sin 105°} = 896.57\ N$$

$T_{OA} = 732\ N$ $\boxed{R_B = 896.57\ N}$

(P) Two identical rollers each of 50N, are supported by an inclined plane and a vertical wall as shown. Find the reactions at the points of supports A, B and C. Assume all surface are smooth.

Sln:

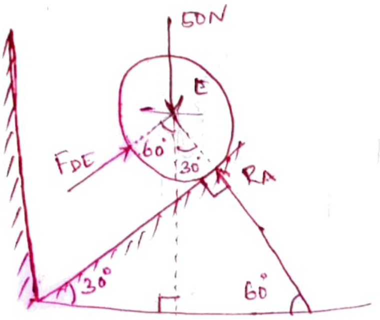


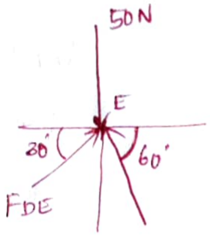


Sln:

Freebody diagram of roller (2).

The angles of F_{DE} and R_A with horizontal are 30° and 60° resp.





Now applying the equation of equilibrium at E

$\Sigma H = 0 \Rightarrow F_{DE} \cos 30° - R_A \cos 60° = 0 \rightarrow (1)$

$\Sigma V = 0 \Rightarrow F_{DE} \sin 30° + R_A \sin 60° - 50 = 0 \rightarrow (2)$

Solving (2) & (1)

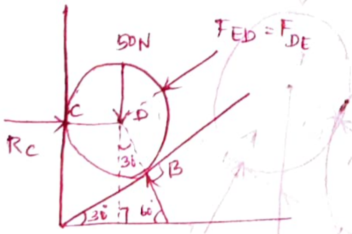
$F_{DE} \cos 30° = R_A \cos 60°$

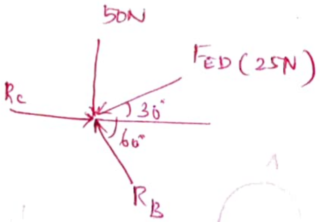
$F_{DE} = 0.577 R_A \Rightarrow$ in (2)

$(0.577 R_A \times 0.5) + (0.866 R_A) = 50$

$1.154 R_A = 50$ $\boxed{R_A = 43.32 N}$ $F_{DE} = 25 N.$

Free body diagram of roller (1).





$F_{ED} = F_{DE} = 25 N$

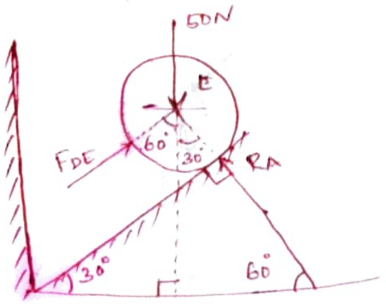
$\Sigma H = 0 \Rightarrow R_C - 25 \cos 30° - R_B \cos 60° = 0 \rightarrow (1)$

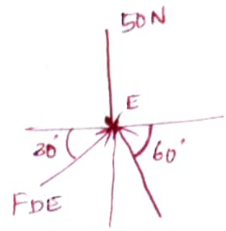
$\Sigma V = 0 \Rightarrow R_B \sin 60° - 25 \sin 30° - 50 = 0 \rightarrow (2)$

$R_B \sin 60° = 50 + 25 \sin 30°$

$R_B \sin 60° = 62.5$

$\boxed{R_B = 72.17 N.}$





Now applying the equations of equilibrium at E

$\Sigma H = 0 \Rightarrow F_{DE} \cos 30° - R_A \cos 60° = 0 \rightarrow (1)$

$\Sigma V = 0 \Rightarrow F_{DE} \sin 30° + R_A \sin 60° - 50 = 0 \rightarrow (2)$

Solving (2) & (1)

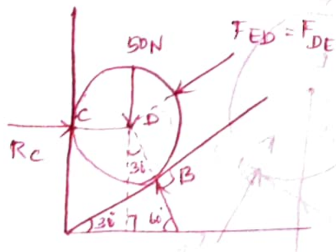
$F_{DE} \cos 30° = R_A \cos 60°$

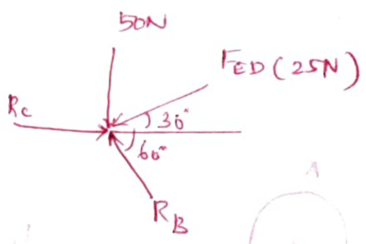
$F_{DE} = 0.577 R_A \Rightarrow$ in (2)

$(0.577 R_A \times 0.5) + (0.866 R_A) = 50$

$1.154 R_A = 50$ $\boxed{R_A = 43.32\,N}$ $F_{DE} = 25\,N.$

Free body diagram of roller (1).





$F_{ED} = F_{DE} = 25\,N$

$\Sigma_H = 0 \Rightarrow R_C - 25 \cos 30° - R_B \cos 60° = 0 \rightarrow (1)$

$\Sigma_V = 0 \Rightarrow R_B \sin 60° - 25 \sin 30° - 50 = 0 \rightarrow (2)$

$R_B \sin 60° = 50 + 25 \sin 30°$

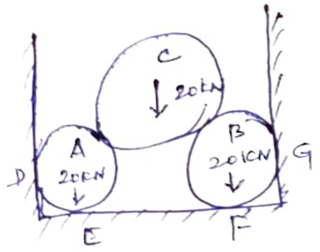
$R_B \sin 60° = 62.5$

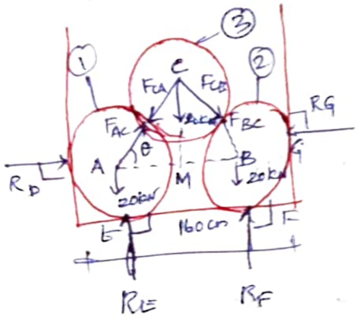
$\boxed{R_B = 72.17\,N.}$

$R_C - 25\cos 30^\circ - 72.17\cos 60^\circ = 0$

$$\boxed{R_C = 57.73\,N}$$

(P) 3 smooth pipes each weighing 20KN and of diameter 60cm are to be placed in a rectangular channel with horizontal base as shown. Calculate the reactions at the point of contact between the pipes and between the channel and pipes. Take the width of channel as 160 cm.





In ΔABC

$DA = BG = \text{radius}$

$= 30cm$

side $AB = 160 - DA - BG$

$= 160 - 30 - 30 = 100\,cm$

$AC = BC = 2 \times r = 2 \times 30 = 60\,cm$.

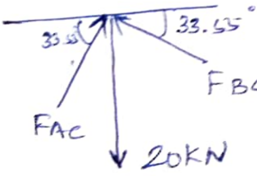
Draw a vertical line through C, $\perp r$ to AB, to intersect at M

ΔBMC

$$BM = \frac{AB}{2} = \frac{100}{2} = 50\,cm.$$

$$\cos\theta = \frac{BM}{BC} = \frac{50}{60} \Rightarrow \theta = 33.55^\circ$$

Free body diagram of pipe (3)



$\Sigma H = 0$

$F_{AC}\cos 33.55 - F_{BC}\cos 33.55 = 0$

$F_{AC} = F_{BC} \rightarrow (1)$

$\Sigma V = 0$

$F_{AC}\sin 33.55 + F_{BC}\sin 33.55 - 20 = 0 \rightarrow (2)$

sub $F_{AC} = F_{BC}$ in (2)

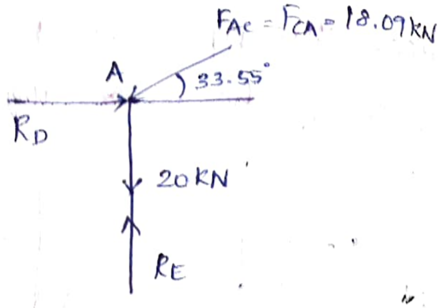
$F_{BC} \sin 33.55 + F_{BC} \sin 33.55 - 20 = 0$

$2 F_B \sin 33.55 = 20$

$F_{BC} = 18.09\,kN$

$F_{AC} = F_{BC} = 18.09\,kN$

Free body diagram of pipe (1).



$\Sigma H = 0$

$\Rightarrow R_D - F_{CA} \cos 33.55 = 0$

$R_D = 18.09 \cos 33.55$

$$\boxed{R_D = 15.07\,kN}$$

$\Sigma V = 0$

$R_E - 20 - F_{CA} \sin 33.55 = 0$

$R_E - 20 - 18.09 \sin 33.55 = 0$

$$\boxed{R_E = 30\,kN}$$

$R_G = R_D = 15.07\,kN \qquad R_F = R_E = 30\,kN$