CENTROID OF COMPOSITE PLANE FIGURES

If plane figure is a combinational of two or more simple plane figures, the algebraic sum of moments of the individual areas about any axis of reference will be equal to the moment of the whole area about the same axis. Hence the centroid of the composite plane figure are determined by the method of moments.

$$\bar{x} = \frac{a_1\ x_1\ + a_2\ x_2\ + \cdots + a_n\ x_n}{a_1\ + a_2\ + a_3 + \cdots + a_n} \quad |||^{ly} \qquad \bar{x} = \frac{a_1\ y_1\ + a_2\ y_2\ + \cdots + a_n\ y_n}{a_1\ + a_2\ + a_3 + \cdots + a_n}$$

a_1, a_2 ... Area of the simple plane figures 1,2 ...

x_1, x_2 ... Horizontal distance of the centroid of simple plane figures 1,2 ... from the horizontal ref axis $0y$..

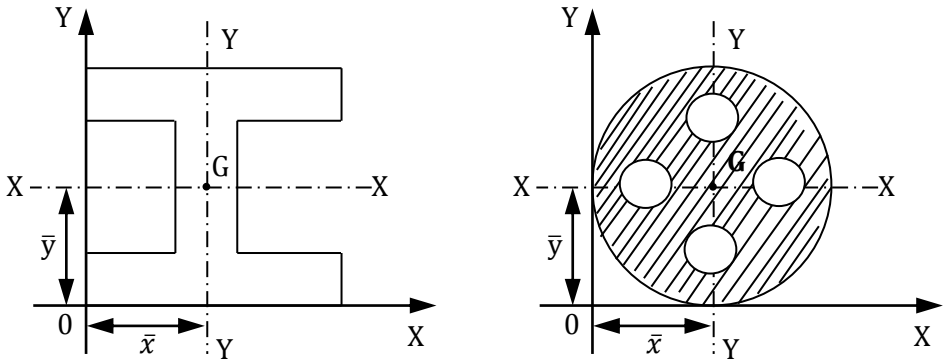
y_1, y_2 ... Vertical distance of the centroid of simple plane figures 1,2 ... from the horizontal ref axis $0x$.

$\bar{x}$ and $\bar{y}$ Horizontal and vertical distance of the centroid of the composite plane figure from the vertical and horizontal reference axis

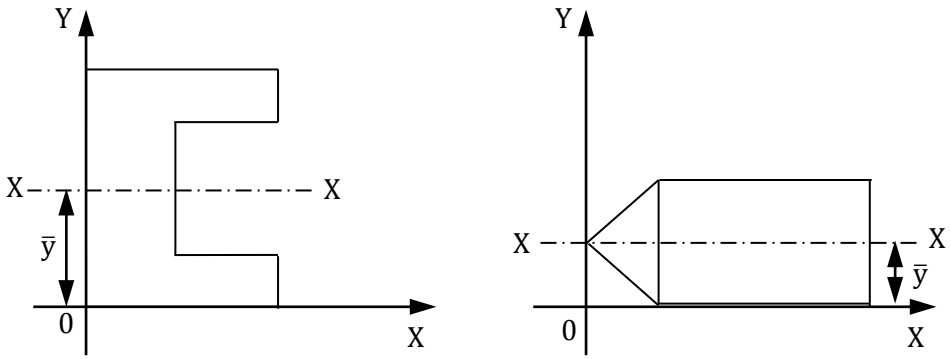
AXIS OF SYMMETRY

An axis about which similar configuration is seen on either side.

(i) Symmetrical About Both The Axes

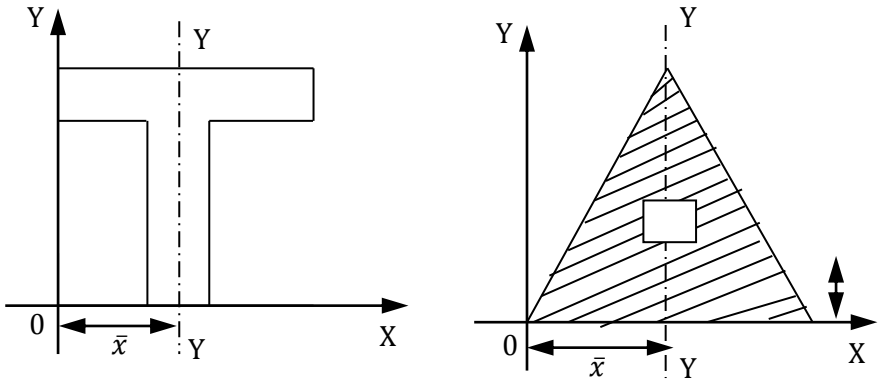


(ii) Symmetrical About X Axis



The centroid lies on the axis of symmetry. Hence no calculation is required for $\bar{y}$, but $\bar{x}$ value is to be found out.

(iii) Symmetrical About Y Axis

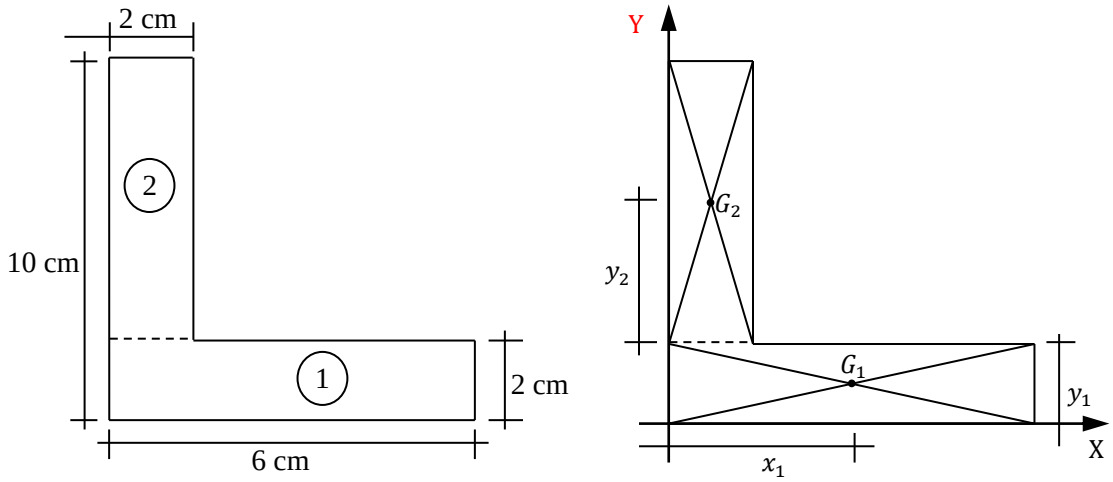


From the geometry $\bar{x}$ value can be determined directly, but calculation is required for finding $\bar{y}$.

(iv) Not Symmetrical About Any Axis

Calculation required for both $\bar{x}$ and $\bar{y}$

Problem 1: locate the centroid of the L section shown in fig.



Solution:

Portion 1: (size 6cm × 2cm)

Area $a_1 = 6 \times 2 = 12cm^2$

Horizontal distance of centroid G_1 from 0Y axis, $x_1 = \frac{6}{2} = 3cm.$

Vertical distance of centroid G_2 from 0X axis, $y_2 = \frac{2}{2} = 1cm.$

Portion 2: (size 2cm × 8cm)

Area $a_2 = 2 \times 8 = 16cm^2$

Horizontal distance of centroid G_2 from 0Y axis, $x_2 = \frac{2}{2} = 1cm.$

Vertical distance of centroid G_2 from 0X axis, $y_2 = \frac{8}{2} = 4cm.$

Using the relation $\bar{x} = \frac{a_1x_1+a_2x_2}{a_1+a_2} = \frac{(12\times3)+(16\times1)}{12+16} = 1.857cm$

And $\bar{y} = \frac{a_1y_1+a_2y_2}{a_1+a_2} = \frac{(12\times1)+(16\times4)}{12+16} = 2.714cm$