MOMENT OF INERTIA:

The moments of a force about a point is the product of the force (F) and the perpendicular distance (x) between the point and the line of action of the force $m_{o_1} = F \times x$

This moment is called as the first moment of force.

If this moment is again multiplied by perpendicular distance (x) between the point and line of action of the force i.e, $m_{o_2} = F \times x = Fx^2$

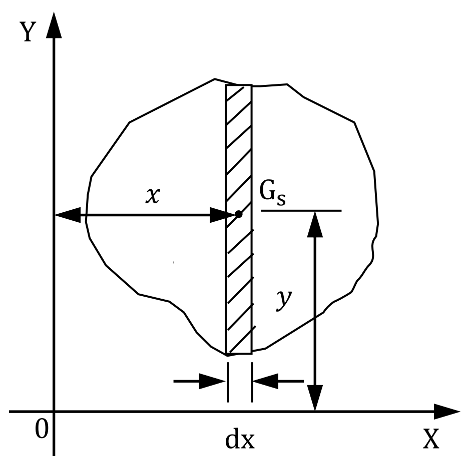
Then this quantity is called as moment of the moment of a force or second moment of force.

The second moment is called as moment of inertia.

Instead of force .the area of a body is taken into consideration, then the second moment is known as the moment of inertia m.I of an area.

m.I of plane figures:

Consider a plane figure whose moment of inertia is required about 0X and 0Y axis as shown in fig.



m.I is denoted by I and carries with it the symbol of the axis about which it is calculated.

m.I of the strip about 0Y axis, I_{0Y} srip= $area \times distance^2$

$$= dA \times x^2$$

m.I of the strip about 0X axis

I_{0X} srip= $area \times distance^2$

$$= dA \times y^2$$

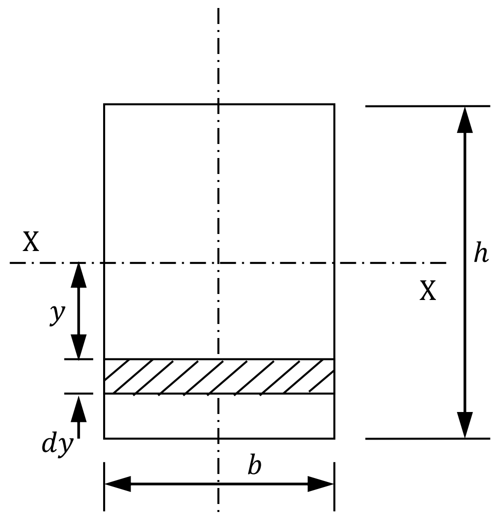
$$\therefore I_{0X} = \int I_{0X} Strip\ dy$$

$$I_{0Y} = \int I_{0Y}\ Strip\ dx$$

$$I_{0X} = \sum dA \cdot y^2$$

$$I_{0Y} = \sum dA \cdot x^2$$

Moment of Inertia of rectangular section:



Consider a rectangle of base 'b' and height 'h' as shown in fig. Let us find the m.I of this rectangle about its centroidal axis XX and YY.

Moment of inertia about XX axis:

Consider a strip AB. parallel to XX axis, base 'b' and thickness 'dy' as shown in fig. Let the distance of this strip from XX axis 'Y'.

Area of the strip= b.dy

m.I of the strip about XX axis=$Area \times distance^2$

$(I_{XX})Strip = (b.dy)y^2 = by^2dy$----- 1

To find the m.I of the whole section about XX axis, integrate the result 1 1 n the limits –h/2 to h/2

$$\therefore (I_{XX})\ Rectangle = \int_{-\mathrm{h}/2}^{\mathrm{h}/2} (I_{XX}) Strip$$

$$= \int_{-\mathrm{h}/2}^{\mathrm{h}/2} by^2 dy = b\left[\frac{y^3}{3}\right]_{-\frac{h}{2}}^{\frac{h}{2}}$$

$$= \frac{b}{3}\left[\left(\frac{h}{2}\right)^2 - \left(-\frac{h}{2}\right)^3\right]$$

$$= \frac{b}{3}\left[\frac{h^3}{8} + \frac{h^3}{8}\right] = \frac{b}{3}\left(\frac{2h^3}{8}\right)$$

$$I_{XX} = \frac{bh^3}{12}$$

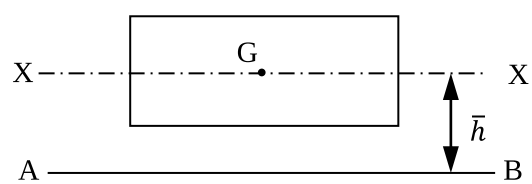
Similarly we can prove

$$I_{YY} = \frac{hb^3}{12}$$

Parallel axis theorem:

It states that "the moment of inertia of a lamina about any axis in the plane of lamina is equal to the sum of the moment of inertia about a parallel centroidal axis in the lamina and square of the distance between the two axes."

m.I of a rectangle about its horizontal centroidal axis



$$I_{XX} = \frac{bh^3}{12}$$

The moment of inertia of the same rectangle, about any axis, but parallel to XX axis can be determined by parallel axis theorem. Let AB is a parallel axis , parallel to XX, at a distance of $\bar{h}$. From parallel axis theorem

$$I_{AB} = I_{XX} + A\bar{h}^2$$

Using the above result, the m.I of the rectangle about its bottom edge can be determined as below

$$I_{AB} = I_{XX} + \left(A\bar{h}\right)^2 \quad A = (b \times h)$$

$$= \frac{bh^3}{12}\left[bh \times \left(\frac{h}{2}\right)^2\right]$$

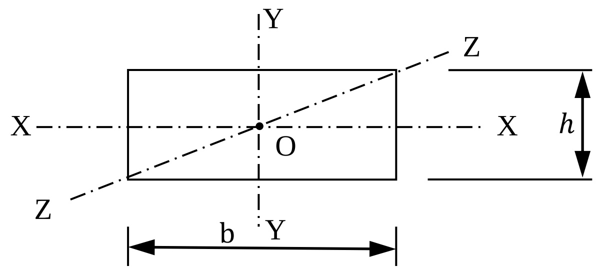
$$= \frac{bh^3}{12} + \frac{bh^3}{4} = \frac{4bh^3}{12} = \frac{bh^3}{3}$$

Perpendicular axis theorem:

It states that “If I_{0X} and I_{0Y} be the moment of inertia of a lamina about two mutually perpendicular axes 0X and 0Y in the plane of the lamina and I_{0Z} be the moment of Inertia of the lamina about an axis normal to the lamina and passing through the point of intersection of the axes 0X and 0Y,then

$$I_{0Z} = I_{0X} + I_{0Y}$$

Consider a rectangle of base ’b’ and height ‘h’ as shown in fig.



We know

$$I_{XX} = \frac{bh^3}{12}$$

and

$$I_{YY} = \frac{hb^3}{12}$$

From perpendicular axis theorem, m.I of the rectangle about axis ZZ, passing through the point of intersection of XX and YY axes and normal to the plane of rectangle

$$I_{ZZ} = I_{XX} + I_{YY} = \frac{bh^3}{12} + \frac{bh^3}{12} = \frac{1}{12}[bh^3 + bh^3]$$