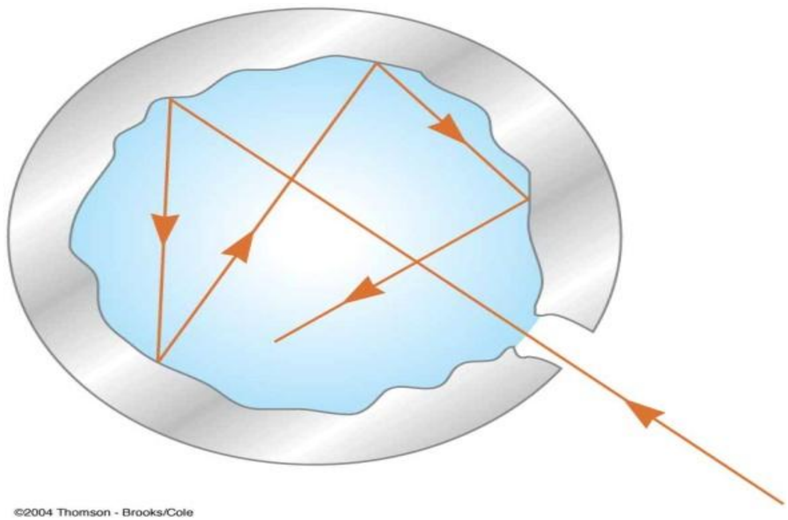
BLACK BODY RADIATION

- **Blackbody**
- a cavity, such as a metal box with a small hole drilled on to it.
- Incoming radiations entering the hole keep bouncing around inside the box with a negligible change of escaping again through the hole => Absorbed , the hole is the ***perfect absorber***



- When heated, it would emit more radiations from a unit area through the hole at a given temperature => ***perfect emitter***

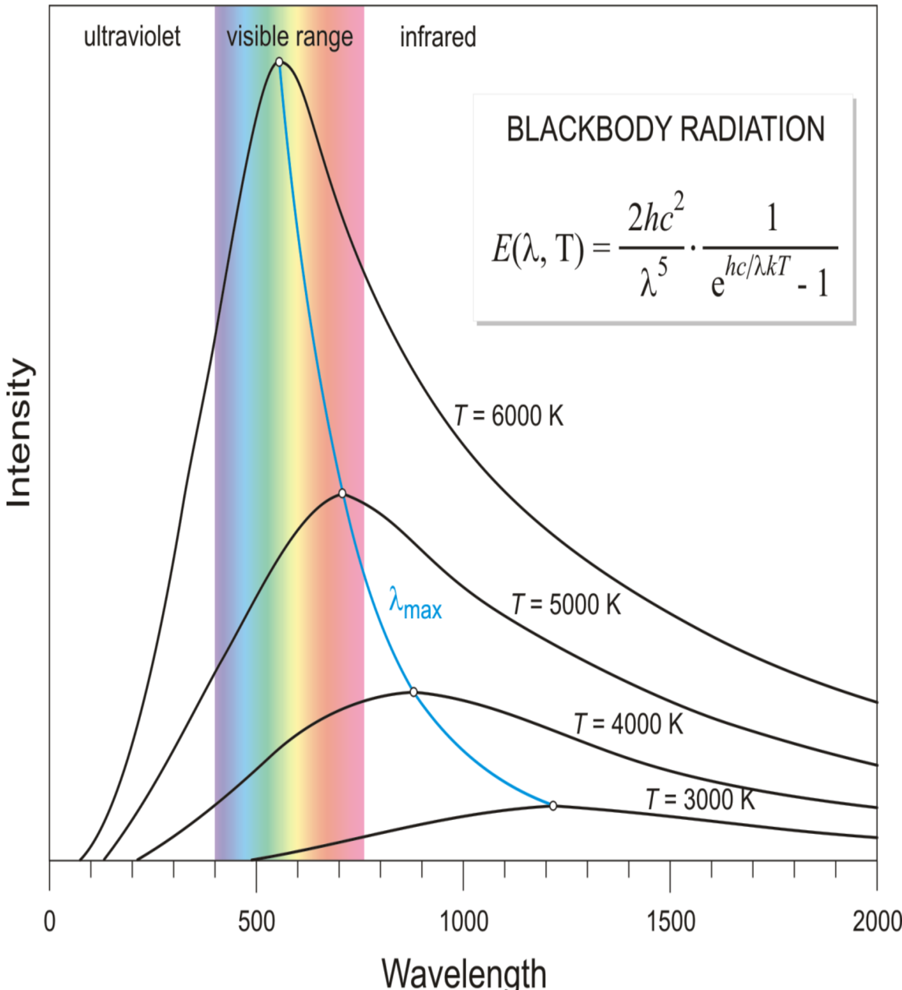
BLACK BODY RADIATION

- A perfect Black body absorbs radiation of all wavelength incident on it. It also emits radiation of all wavelength.
- When Black body is at a higher temperature than its surrounding, then emission is more than absorption.
- The heat radiation emitted by a black body is known as Black body radiation.

Theory of Black Body Radiation-Energy spectrum of a black body

The radiation emitted by the black body varies with temperature.

- At a given temp. energy distribution is not continuous.
- The intensity of radiation is maximum at a particular wavelength λ_{max}.
- With increase in temperature λ_{max} decreases
- As temp. Increases intensity also increases.
- The area under each spectrum represents the total energy emitted at that particular temp.



Basic definitions

Stefan's Boltzmann's law:

- Total energy emitted at a particular temp.of a object is directly proportional to the fourth power of the temperature of the body. $E \propto T^4$

Wien's displacement law:

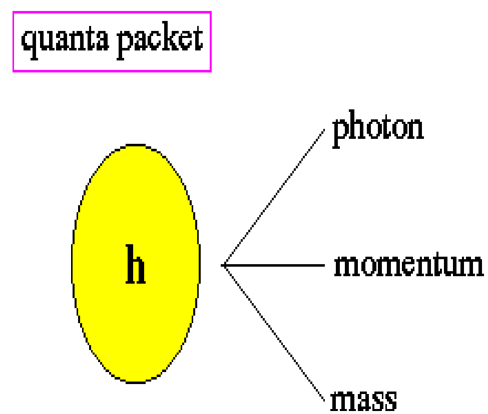
The product of wavelength corresponding to maximum intensity λ_{max} and absolute temp. in a hot body is a constant. $\lambda_{max} T = \text{Cons.}$

Rayleigh-Jeans law:

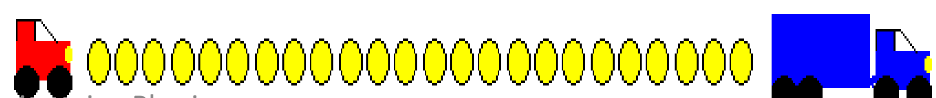
Energy distribution is directly proportional to Absolute temp. and inversely proportional to fourth power of wavelength of radiation of hot body.

$$E_\lambda = \frac{8\pi KT}{\lambda^4}$$

PLANK'S THEORY



PLANK'S THEORY



PLANCK'S QUANTUM THEORY OF BLACK BODY RADIATION

Planck's theory: [Hypothesis]

A black body contains a **large number of oscillating particles**:

Each particle is vibrating with a characteristic frequency.

The frequency of radiation emitted by the oscillator is the same as the oscillator **frequency.**

The oscillator can absorb energy in multiples of small unit called quanta.

This **quantum of radiation** is **photon.**

The energy of a **photon** is directly proportional to the **frequency of radiation** emitted.

An oscillator **vibrating with frequency** can only emit energy in integral multiples of $h\nu$. , where n= 1, 2, 3, 4…….n. n is **quantum number**.

PLANCK'S LAW OF RADIATION

The energy density of radiations emitted by a black body at a temperature T in the **wavelength** range λ **to λ+dλ** is

$$E_\lambda d\lambda = \frac{8\pi hC}{\lambda^5[\exp\left(\frac{hC}{\lambda KT}\right)-1]}d\lambda$$

h =6.625x10^{-34}Js^{-1} - Planck's constant.

C= 3x10^{8} m/s -velocity of light

K = 1.38x 10^{-23} J/K -Blotzmann constant

T is the absolute temperature in kelvin.

DERIVATION OF PLANCK'S LAW

Consider a black body with a large number of atomic oscillators. Average energy per oscillator is

$$\overline{E} = \frac{E}{N} - - - - - (1)$$

E is the total energy of all the oscillators and N is the number of oscillators.

Let the number of oscillators in ground state is N_0.

According to Maxwell's law of distribution, the number of oscillators having an energy value E_N is

$$N_n = N_0 e^{-\frac{E_n}{kT}} - - - - - (2)$$

DERIVATION OF PLANCK'S LAW

T is the absolute temperature. K is the Boltzmann constant.

Let N_0 be the number of oscillators having energy E_0,

- N_1 be the number of oscillators having energy E_1,
- N_2 be the number of oscillators having energy E_2 and so on.

Then

$$N = N_0 + N_1 + N_2 + \ldots\ldots(3)$$

$$N = N_0 + N_0 e^{\frac{-E_1}{KT}} + N_0 e^{\frac{-E_2}{KT}} + \ldots\ldots(4)$$

From Planck's theory, E can take only integral values of $h\nu$.
Hence the possible energy are 0, $h\nu$, $2h\nu$, $3h\nu$……. and so on.

$$i.e.\ E_n = nh\nu, where\ n = 0, 1, 2, 3$$

DERIVATION OF PLANCK'S LAW

$$E_0 = 0, \qquad E_1 = h\nu, \qquad E_2 = 2h\nu, \qquad E_3 = 3h\nu,$$

$$N = N_0 + N_0 e^{\frac{-h\nu}{KT}} + N_0 e^{\frac{-2h\nu}{KT}} + \ldots..(5)$$

Taking $x = e^{-\frac{h\nu}{kT}}$ $in\ (5)$

$$N = N_0 + N_0 x + N_0 x^2 + N_0 x^3 \ldots..(6)$$

$$N = N_0 [1 + x + x^2 + x^3 \ldots.].$$

$$N = \frac{N_0}{(1-x)} \ldots\ldots\ldots\ldots 7)$$ **(** using Binomial expansion**)**

DERIVATION OF PLANCK'S LAW

The total energy

$$E = E_0 \ N_0 + E_1 \ N_1 + E_2 \ N_2 + E_3 \ N_3 \ldots (8)$$

Substituting the value of $E_0 \ E_1 \ E_2 \ E_3 \ etc$

$$E = 0XN_0 + h\nu N_0 e^{-\frac{h\nu}{kT}} + 2h\nu N_0 e^{-\frac{2h\nu}{kT}} + 3h\nu N_0 e^{-\frac{3h\nu}{kT}} \ldots. +$$

$$E = h\nu \, N_0 e^{-\frac{h\nu}{kT}} + 2h\nu \, N_0 e^{-\frac{2h\nu}{kT}} + \cdots . \pm - - - - - - (9)$$

Putting

$$x = e^{-\frac{h\nu}{kT}} \ in \ (8)$$

$$E = h\nu \ N_0 x + 2h\nu \ N_0 x^2 + \cdots . \pm - - - (10)$$

DERIVATION OF PLANCK'S LAW

$$E = h\nu N_0[x + 2x^2 + \ \ldots +]$$

$$E = h\nu N_0 x[1 + 2x + \ \ldots +]$$

$$E = \frac{h\nu\, N_0 x}{(1-x)^2} - - - - - - (11)$$

$$Since\ \left\{\frac{1}{(1-x)^2} = (1-x)^{-2} = 1 + 2x + \cdots\right\}$$

Substituting (11) and (7) in (1)

$$\overline{E} = \frac{\left(\frac{h\nu N_0 x}{(1-x)^2}\right)}{\frac{N_0}{(1-x)}}$$

DERIVATION OF PLANCK'S LAW

$$\overline{E} = \frac{h\nu x}{(1-x)} \qquad \overline{E} = \frac{h\nu x}{x\left(\frac{1}{x}-1\right)} \qquad \overline{E} = \frac{h\nu}{\left(\frac{1}{x}-1\right)}$$

Substituting the value for x

$$\overline{E} = \frac{h\nu}{\left(\frac{1}{e^{-\frac{h\nu}{kT}}}-1\right)}$$

$$\overline{E} = \frac{h\nu}{\left(e^{\frac{h\nu}{kT}}-1\right)} ----(12)$$

The number of oscillators per unit volume in the wavelength range λ and $\lambda+d\lambda$ is

$$\frac{8\pi d\lambda}{\lambda^4} -------(13)$$

DERIVATION OF PLANCK'S LAW

Hence the energy density of radiation between the wavelength range $\lambda$and λ+dλ is

E_λ dλ = No. of oscillator per unit volume in the range λ and λ+dλ X Average energy.

$$E_\lambda d\lambda = \frac{8\pi d\lambda}{\lambda^4} X \frac{h\nu}{\left(e^{\frac{h\nu}{kT}} - 1\right)} - - - - -(1)$$

$$E_\lambda d\lambda = \frac{8\pi d\lambda}{\lambda^4} \frac{hC/\lambda}{\left(e^{\frac{h\nu}{kT}} - 1\right)}$$

$$E_\lambda d\lambda = \frac{8\pi d\lambda}{\lambda^5} \frac{hC}{\left(e^{\frac{h\nu}{kT}} - 1\right)}$$

$$E_\lambda d\lambda = \frac{8\pi d\lambda}{\lambda^5} \frac{hC}{\left(e^{\frac{h\nu}{kT}} - 1\right)}$$

DERIVATION OF PLANCK'S LAW

$$E_{\lambda} d\lambda = \frac{8\pi hC}{\lambda^5} \frac{1}{\left(e^{\frac{h\nu}{kT}} - 1\right)} d\lambda - - - - - (15)$$

$$E_{\lambda} d\lambda = \frac{8\pi hC}{\lambda^5 [e^{\left(\frac{hC}{\lambda KT}\right)} - 1]} d\lambda$$

The equation (16) represents **Planck's law of radiation.**

Planck's law can also be represented in **terms of frequencies.**

$$E_{\nu} d\nu = \frac{8\pi h \nu^3}{C^3} \frac{1}{\left(e^{\frac{h\nu}{kT}} - 1\right)} d\nu - - - - - - - 17$$