



DEPARTMENT OF MATHEMATICS

UNIT-IV APPLICATION OF PARTIAL DIFFERENTIAL EQUATION

TYPE-II Heat flux in y direction, i.e. $0 < y < l$.

The boundary conditions are:

$$(i) u(x, 0) = 0$$

$$(ii) u(x, l) = 0$$

$$(iii) u(0, y) = 0$$

$$(iv) u(l, y) = f(y), 0 < y < l.$$

The suitable soln. is $u(x, y) = (A e^{Px} + B e^{-Px})(C \cos py + D \sin py)$

) A square plate bld. by the lines $x=0$, $y=0$, $x=b$ & $y=b$ its faces are insulated. The temp. along the lower vertical edges is gn. by $u(b, y) = y(b-y)$, $0 < y < b$, while the other three edges kept at 0° . Find the steady state temp. in the plate.

Soln: The boundary condns. are

$$(i) u(x, 0) = 0$$

$$(ii) u(x, b) = 0$$

$$(iii) u(0, y) = 0$$

$$(iv) u(b, y) = y(b-y)$$

The suitable soln. is

$$u(x, y) = (A e^{Px} + B e^{-Px})(C \cos py + D \sin py)$$

Apply (i) $u(x, 0) = (A e^{Px} + B e^{-Px})C$

$$\boxed{0 = C}$$



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$$\therefore u(x, y) = (A e^{px} + B e^{-px}) D \sin py .$$

Apply (ii), $u(0, b) = (A e^{pn} + B e^{-pn}) D \sin pb .$

$$0 = \sin pb D (A e^{pn} + B e^{-pn})$$

$$\Rightarrow D \neq 0, \sin pb = 0$$

$$p = \frac{n\pi}{b}$$

$$\therefore u(x, y) = (A e^{\frac{n\pi}{b}x} + B e^{-\frac{n\pi}{b}x}) D \sin \frac{n\pi}{b}y .$$

Apply (iii), $u(0, y) = (A + B) D \sin \frac{n\pi}{b}y .$

$$0 = A + B \Rightarrow [B = -A]$$

$$\therefore u(x, y) = AD \left(e^{\frac{n\pi}{b}x} - e^{-\frac{n\pi}{b}x} \right) \sin \frac{n\pi}{b}y .$$

General Soln. $u(x, y) = \sum_{n=1}^{\infty} A_n \sin \frac{n\pi}{b}x \sin \frac{n\pi}{b}y .$

$$y(b-y) = \sum_{n=1}^{\infty} A_n \sin \frac{n\pi}{b}x \sin \frac{n\pi}{b}y .$$

$$y(b-y) = \sum_{n=1}^{\infty} B_n \sin \frac{n\pi}{b}y, \quad B_n = A_n \sin \frac{n\pi}{b}x .$$

$$B_n = \frac{2}{b} \int_0^b y(b-y) \sin \frac{n\pi}{b}y dy .$$

$$= \frac{4l^2}{n^2\pi^2} [1 - (-1)^n] .$$



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$$A_n = \frac{4l^2}{n^2\pi^2} \left[\frac{1 - (-1)^n}{\sin nh n\pi} \right]$$

$$\therefore u(x,y) = \sum_{n=1}^{\infty} \frac{4l^2}{n^2\pi^2} \left[\frac{1 - (-1)^n}{\sin nh n\pi} \right] \frac{\sin \frac{nh n\pi}{b}}{b} x \sin \frac{n\pi}{b} y$$